SUPPLEMENT

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CITY AND GUILDS OF LONDON INSTITUTE EXAMINATIONS, 1967-68

QUESTIONS AND ANSWERS

Answers are occasionally omitted or reference is made to earlier Supplements in which questions of substantially the same form, together with the answers, have been published. Some answers contain more detail than would be expected from candidates under examination conditions.

MATHEMATICS B, 1968

Students were expected to answer any six questions

Q. **1.** *The number of trunks (N) required for a given amount of telephone traffic (A traffic units) is quoted in three instances as:*

A	0·9	4	10
N	5	11	20

Assuming a working formula

$$N = pA + qA^{1/2} + rA^{1/3},$$

calculate the constants p, q and r to 2 decimal places.

A. 1. $N = pA + qA^{1/2} + rA^{1/3}$.

Substituting each pair of values of A and N in the above, the following three equations are obtained:

$$5 = 0{\cdot}9p + 0{\cdot}9^{1/2}q + 0{\cdot}9^{1/3}r,$$
$$\text{or, } 5 = 0{\cdot}9p + 0{\cdot}9487q + 0{\cdot}9655r. \quad \ldots\ldots (1)$$

$$11 = 4p + 4^{1/2}q + 4^{1/3}r,$$
$$\text{or, } 11 = 4p + 2q + 1{\cdot}587r. \quad \ldots\ldots (2)$$

$$20 = 10p + 10^{1/2}q + 10^{1/3}r,$$
$$\text{or, } 20 = 10p + 3{\cdot}162q + 2{\cdot}154r. \quad \ldots\ldots (3)$$

Note. The values of $A^{1/2}$ may be obtained from a table of square roots whilst those of $A^{1/3}$ may be obtained direct from the values of $\sqrt[3]{n}$ or $\sqrt[3]{10n}$ given in most books of 4-figure logarithmic and other tables. If the latter tables are not available, the required values are easily deducible from logarithms.

Multiplying equation (1) by $\frac{4}{0{\cdot}9}$ gives:

$$22{\cdot}222 = 4p + 4{\cdot}216q + 4{\cdot}29r. \quad \ldots\ldots (4)$$

No.	*Log.*
4·4	0·6478
0·9487	$\bar{1}$·9771+
4·216	0·6249
4·4	0·6478
0·9655	$\bar{1}$·9847+
4·29	0·6325

Subtracting equation (2) from equation (4) gives:

$$11{\cdot}222 = 2{\cdot}216q + 2{\cdot}703r. \quad \ldots\ldots (5)$$

Multiplying equation (2) by $\frac{5}{2}$ gives:

$$27{\cdot}5 = 10p + 5q + 3{\cdot}9675r. \quad \ldots\ldots (6)$$

Subtracting equation (3) from equation (6) gives:

$$7{\cdot}5 = 1{\cdot}838q + 1{\cdot}8135r. \quad \ldots\ldots (7)$$

Multiplying equation (5) by $\frac{1{\cdot}838}{2{\cdot}216}$ gives:

$$11{\cdot}222 \times \frac{1{\cdot}838}{2{\cdot}216} = 1{\cdot}838q + 2{\cdot}703 \times \frac{1{\cdot}838}{2{\cdot}216}r.$$

$$\text{or, } 9{\cdot}307 = 1{\cdot}838q + 2{\cdot}243r \quad \ldots\ldots (8)$$

Subtracting equation (7) from equation (8) gives:

$$1{\cdot}807 = 0{\cdot}4295r,$$
$$\text{or, } r = \frac{1{\cdot}807}{0{\cdot}4295},$$
$$= 4{\cdot}207.$$

From equation (7),

$$1{\cdot}838q = 7{\cdot}5 - 1{\cdot}8135r.$$

Substituting for r in this equation gives:

$$1{\cdot}838q = 7{\cdot}5 - 1{\cdot}8135 \times 4{\cdot}207,$$
$$= 7{\cdot}5 - 7{\cdot}63,$$
$$= -0{\cdot}13.$$
$$\therefore q = -\frac{0{\cdot}13}{1{\cdot}838},$$
$$= -0{\cdot}07071.$$

From equation (2),

$$4p = 11 - 2q - 1{\cdot}587r.$$

No.	*Log.*
11·222	1·0500
1·838	0·2644+
	1·3144
2·216	0·3456−
9·307	0·9688
2·703	0·4319
1·838	0·2644+
	0·6963
2·216	0·3456−
2·243	0·3507
1·807	0·2570
0·4295	$\bar{1}$·6330−
4·207	0·6240
1·8135	0·2585
4·207	0·6240+
	0·8825
0·13	$\bar{1}$·1139
1·838	0·2644−
	$\bar{2}$·8495
1·587	0·2006
4·207	0·6240+
	0·8246

Substituting for q and r in this equation gives:

$$4p = 11 - 2 \times (-0{\cdot}07071) - 1{\cdot}587 \times 4{\cdot}207,$$
$$= 11 + 0{\cdot}14142 - 6{\cdot}677,$$
$$= 11{\cdot}1414 - 6{\cdot}677.$$
$$\therefore p = \frac{4{\cdot}4644}{4},$$
$$= 1{\cdot}1161.$$

Therefore, to 2 decimal places,

$$\underline{p = 1{\cdot}12} \quad \underline{q = -0{\cdot}07} \quad \underline{r = 4{\cdot}21}.$$

Q. **2.** *The current i mA in a circuit at a time t ms from the closure of a switch is given by*

$$i = 15(1 - e^{-0{\cdot}4t})$$

Derive an expression for t in terms of i.
Plot the graph of i from t = 0 to t = 6. From the graph, estimate when i is 50 per cent of its ultimate value. Verify your result by calculation.

A. 2.

$$i = 15(1 - e^{-0{\cdot}4t}),$$
$$= 15 - 15e^{-0{\cdot}4t}.$$

$$\therefore\ 15e^{-0\cdot4t} = 15 - i,$$

$$\text{or, } e^{-0\cdot4t} = \frac{15 - i}{15}.$$

Taking logarithms to base e gives:

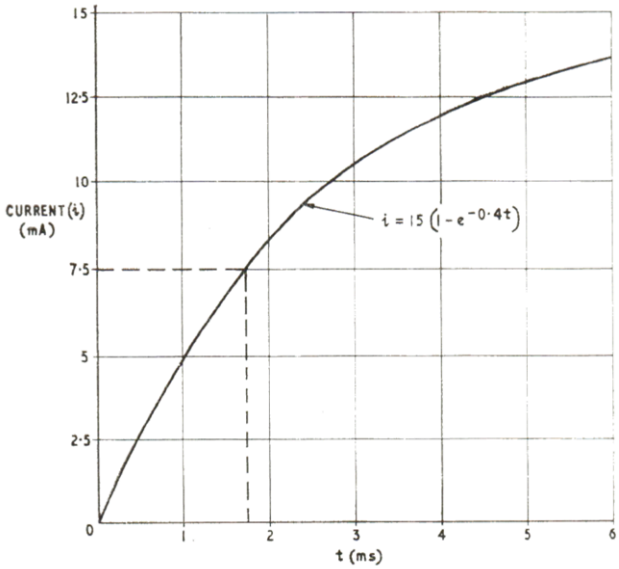
$$-0\cdot4t = \log_e\left(\frac{15 - i}{15}\right),$$

$$\text{or, } t = \underline{-2\cdot5\log_e\left(\frac{15 - i}{15}\right)}.$$

The graph may be plotted from the following table of values:

t	0	1	2	3	4	5	6
$-0\cdot4t$	0	$-0\cdot4$	$-0\cdot8$	$-1\cdot2$	$-1\cdot6$	-2	$-2\cdot4$
$e^{-0\cdot4t}$	1	0·670	0·449	0·301	0·202	0·135	0·091
$1 - e^{-0\cdot4t}$	0	0·330	0·551	0·699	0·798	0·865	0·909
$i = 15(1 - e^{-0\cdot4t})$	0	4·950	8·265	10·485	11·97	12·98	13·635

The graph is shown in the sketch.



The ultimate value of i occurs when $t = \infty$. At this value $e^{-0\cdot4t}$ is zero and hence $i = 15$ mA.

Therefore, half the ultimate value of i is 7·5 mA. From the graph, this occurs when $t = \underline{1\cdot733 \text{ ms}}$.

From the expression for t obtained in the first part of the answer, with $i = 7\cdot5$ mA.,

$$t = -2\cdot5\log_e\left(\frac{15 - 7\cdot5}{15}\right)\text{ms},$$

$$= -2\cdot5\log_e 0\cdot5,$$

$$= -2\cdot5 \times -0\cdot6932,$$

$$= 1\cdot733 \text{ ms}.$$

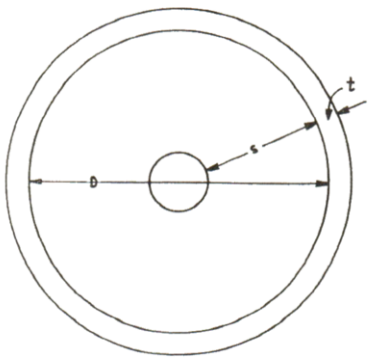
Thus, by calculation, $t = \underline{1\cdot733 \text{ ms}}$ which verifies the value measured from the graph.

Q. **3.** *The outer conductor of a coaxial cable is of internal diameter D and the radial separation between outer and inner conductors is s. The inner conductor is solid and of the same metal as the outer conductor.*

Derive a formula for the thickness t of the outer conductor if it has the same d.c. resistance as the inner conductor.

Calculate t if $D = 1\cdot2$ cm, $s = 0\cdot3$ cm.

A. 3. The sketch shows a cross-section of the coaxial circle of internal diameter D.



The resistance, R, of a uniform length of conductor, l, whose cross-sectional area is A is given by:

$$R = \frac{\rho l}{A},$$

where ρ is the resistivity of the metal forming the conductor.

In comparing resistances of the outer and inner conductors forming the coaxial cable, ρ may be disregarded since it has the same value for either conductor. Also l may similarly be disregarded, since the lengths of inner and outer conductors must be assumed equal.

$$\text{Thus, } R \propto \frac{1}{A}.$$

If the outer conductor of thickness t is to have the same d.c. resistance as the inner conductor, it follows from the above that the cross-sectional areas of both conductors must be the same.

For the inner conductor, the diameter is $(D - 2s)$ from the sketch.

$$\therefore\ A_{inner} = \frac{\pi(D - 2s)^2}{4},$$

$$\text{and, } A_{outer} = \frac{\pi(D + 2t)^2}{4} - \frac{\pi D^2}{4}.$$

Hence, for equal resistances:

$$\frac{\pi(D - 2s)^2}{4} = \frac{\pi(D + 2t)^2}{4} - \frac{\pi D^2}{4},$$

$$\text{or, } (D - 2s)^2 = (D + 2t)^2 - D^2.$$

$$\therefore\ (D + 2t)^2 = (D - 2s)^2 + D^2,$$

$$D^2 + 4Dt + 4t^2 = (D - 2s)^2 + D^2,$$

$$\text{or, } 4t^2 + 4Dt - (D - 2s)^2 = 0.$$

This is a quadratic equation in t and hence, from the general formula for the quadratic:

$$t = \frac{-4D \pm \sqrt{16D^2 + 16(D - 2s)^2}}{8},$$

$$= \frac{-4D \pm 4\sqrt{D^2 + (D - 2s)^2}}{8},$$

$$= \underline{\frac{-D \pm \sqrt{D^2 + (D - 2s)^2}}{2}}.$$

When $D = 1\cdot2$ cm and $s = 0\cdot3$ cm,

$$t = \frac{-1\cdot2 \pm \sqrt{1\cdot44 + (1\cdot2 - 0\cdot6)^2}}{2},$$

$$= \frac{-1\cdot2 \pm \sqrt{1\cdot80}}{2},$$

$$= \frac{-1\cdot2 \pm 1\cdot342}{2},$$

$$= \underline{0\cdot071 \text{ cm}}.$$

Q. **4.** *The table gives the collector current i mA for values of the collector voltage v volts applied to a transistor in a common emitter configuration.*

v	3	5	7	9
i	2·20	2·85	3·65	4·65

Plot suitable variables to verify the formula $i = ae^{kv}$ relating these measurements. From the graph find the constants a and k.

A. 4. $i = ae^{kv}$.

Taking natural logarithms,

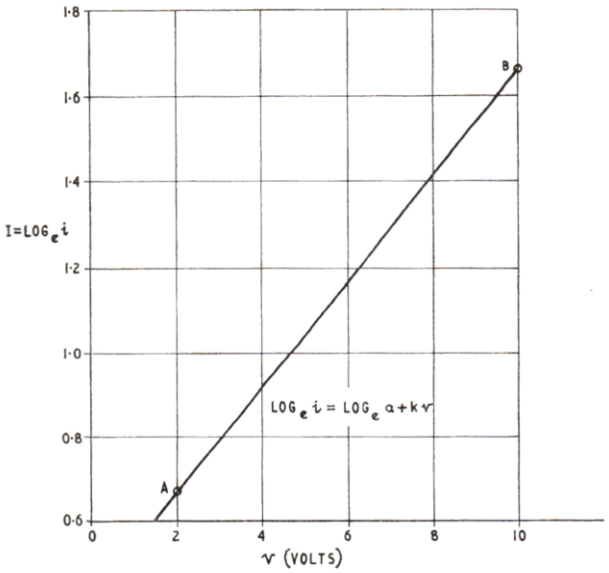
$$\log_e i = \log_e a + kv.$$

Let $\log_e i = I$, and $\log_e a = A$.

Then $I = A + kv$, where I and v are variables whilst A and k are constants. This law is of the linear form $y = ax + b$, and hence, if I plotted against v yields substantially a straight-line graph, the law $i = ae^{kv}$ will be verified. The graph may be plotted from the following table of values:

v	3	5	7	9
i	2·20	2·85	3·65	4·65
$I = \log_e i$	0·7885	1·047	1·295	1·537

The graph is shown in the sketch. The four plotted points lie substantially on a straight line, thus verifying the law $i = ae^{kv}$.



Taking the co-ordinates of two widely-separated points A and B which actually lie on the straight-line graph of the sketch, the gradient, k, is given by,

$$k = \frac{1{\cdot}665 - 0{\cdot}673}{10 - 2},$$

$$= \frac{0{\cdot}992}{8},$$

$$= 0{\cdot}124.$$

The value of A is now obtained by substitution in $I = A + kv$, of the co-ordinates of any point on the graph, say point A, together with the measured value of k.

Thus, $0{\cdot}673 = A + 0{\cdot}124 \times 2$,

or, $A = 0{\cdot}673 - 0{\cdot}248$,

$= 0{\cdot}425$.

But, $A = \log_e a$.

$\therefore\ a = \text{antilog}_e\, 0{\cdot}425$,

$= 1{\cdot}53$.

Thus, $a = \underline{1{\cdot}53}$, and $k = \underline{0{\cdot}124}$.

***Q.* 5.** (*a*) *State the compound angle formulae for* $\cos(A + B)$ *and* $\sin(A + B)$. *Hence derive*

(*i*) $\cos 2\theta$ *in terms of* $\sin\theta$,

(*ii*) *the formula* $\cos\theta - \cos\phi = -2\sin\dfrac{\theta+\phi}{2}\sin\dfrac{\theta-\phi}{2}$.

(*b*) *If* $\dfrac{p}{w} = \dfrac{\sin(\alpha+\lambda)}{\cos\lambda}$,

calculate λ *given* $w = 1{,}000$, $p = 507$, $\alpha = 12°$.

A. 5. (*a*) $\cos(A + B) = \underline{\cos A\cos B - \sin A\sin B.}$

$\sin(A + B) = \underline{\sin A\cos B + \cos A\sin B.}$

(*i*) $\cos 2\theta = \cos(\theta + \theta)$,

$= \cos^2\theta - \sin^2\theta.$ ..... **(1)**

But, $\sin^2\theta + \cos^2\theta = 1$,

or, $\cos^2\theta = 1 - \sin^2\theta.$ ..... (2)

Substituting $\cos^2\theta$ from equation (2) into equation (1) gives

$$\underline{\cos 2\theta = 1 - 2\sin^2\theta.}$$

(*ii*) $\cos(A + B) = \cos A\cos B - \sin A\sin B.$

It may be shown that the above, in common with the other addition theorem, is true for angles of any magnitude. Hence, putting $(-B)$ for B gives,

$\cos(A - B) = \cos A\cos(-B) - \sin A\sin(-B)$.

But, $\cos(-B) = \cos B$,

and, $\sin(-B) = -\sin B$.

$\therefore\ \cos(A - B) = \cos A\cos B + \sin A\sin B$.

Hence, $\cos(A + B) - \cos(A - B) = -2\sin A\sin B.$ ..... (3)

Let $A + B = \theta$, and $A - B = \phi$.

Then, adding θ and ϕ, gives

$$2A = \theta + \phi, \quad \text{or, } A = \frac{\theta+\phi}{2}.$$

Subtracting θ and ϕ gives

$$2B = \theta - \phi, \quad \text{or, } B = \frac{\theta-\phi}{2}.$$

Substituting for A and B in equation (3) gives

$$\underline{\cos\theta - \cos\phi = -2\sin\frac{\theta+\phi}{2}\sin\frac{\theta-\phi}{2}.}$$ Q.E.D.

(*b*) $$\frac{p}{w} = \frac{\sin(\alpha+\lambda)}{\cos\lambda},$$

$$= \frac{\sin\alpha\cos\lambda + \cos\alpha\sin\lambda}{\cos\lambda},$$

$$= \sin\alpha + \cos\alpha\tan\lambda.$$

Substituting the values given in this equation,

$$\sin 12° + \cos 12°\tan\lambda = \frac{507}{1{,}000},$$

or, $0{\cdot}2079 + 0{\cdot}9781\tan\lambda = 0{\cdot}507$.

$\therefore\ 0{\cdot}9781\tan\lambda = 0{\cdot}507 - 0{\cdot}2079$,

$= 0{\cdot}2991$.

or, $\tan\lambda = \dfrac{0{\cdot}2991}{0{\cdot}9781}$.

$\lambda = \underline{17°}$.

No.	*Log.*
0·2991	$\bar{1}$·4758
0·9781	$\bar{1}$·9903 −
tan 17°	$\bar{1}$·4855

***Q.* 6.** (*a*) *The positive numbers a, b, c form a geometrical progression. Prove*

(*i*) $b^2 = ac$,

(*ii*) *log a, log b, and log c form an arithmetical progression.*

(*b*) *State and prove a rule for converting a logarithm to base a into a logarithm to base b.*

Given $\log_{10} 3 = 0{\cdot}4771$, *calculate* $\log_3 10$, *and use this value to find* $\log_3 2{\cdot}7$ *without using mathematical tables.*

A. 6. (*a*) If a, b and c form a geometrical progression then,

(*i*) $b = ar$, and $c = br = ar^2$,

where r is the common ratio.

Thus, $r = \dfrac{b}{a} = \dfrac{c}{b}$,

or, $\underline{b^2 = ac.}$ Q.E.D.

(*ii*) $\log b = \log(ar)$,

$= \log a + \log r$.

$\therefore\ \log b - \log a = \log r.$ ..... (1)

Also, $\log c = \log(br)$,

$= \log b + \log r$.

$\therefore\ \log c - \log b = \log r.$ ..... (2)

Hence, from equations (1) and (2), it can be seen that $\log a$, $\log b$ and $\log c$ form an arithmetical progression whose common difference is $\log r$.

(*b*) Consider a number N and let $\log_a N = p$.
Then by definition of a logarithm:

$$N = a^p.$$
$$\therefore \log_b N = \log_b (a^p),$$
$$= p \log_b a.$$
But, $p = \log_a N.$
$$\therefore \underline{\log_b N = \log_a N \times \log_b a.}$$

Thus, to convert the logarithm of a number N to base a to the logarithm of the same number to base b it is necessary to multiply it by $\log_b a$.

Assuming again that $\log_a N = p$, i.e. $N = a^p$,

then, $\log_N N = \log_N (a^p)$,
$$= p \log_N a.$$
But, $\log_N N = 1$, and $p = \log_a N$.
$$\therefore \log_a N = \frac{1}{\log_N a}.$$
Thus, if $\log_{10} 3 = 0{\cdot}4771$,
$$\log_3 10 = \frac{1}{0{\cdot}4771},$$
$$= \underline{2{\cdot}096.}$$
$$\log_3 2{\cdot}7 = \log_3 \frac{27}{10},$$
$$= \log_3 (3^3) - \log_3 10,$$
$$= 3 - 2{\cdot}096.$$
$$\therefore \log_3 2{\cdot}7 = \underline{0{\cdot}904.}$$

***Q.* 7.** *In Fig.* 1 *the phasors (vectors) I_A, I_B, I_C represent, in magnitude and phase relationship, the r.m.s. currents taken from a three-phase supply.*

The phasor sum of I_A, I_B and I_C gives the r.m.s. current I_R in the common return wire of this three-phase system. Find graphically or otherwise the magnitude of I_R and its phase relative to I_A.

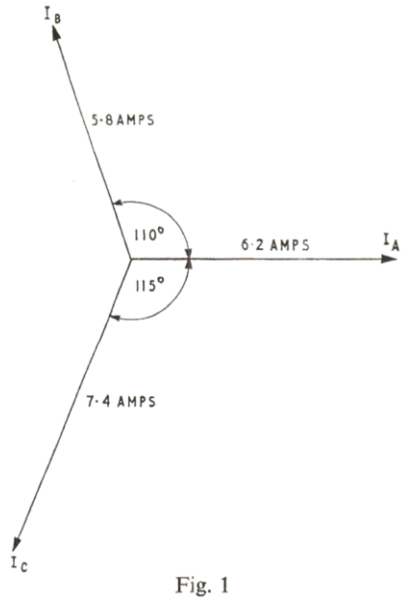


Fig. 1

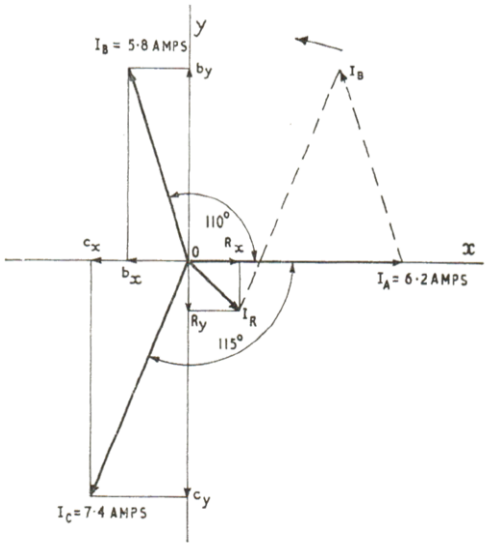
A. 7. The vectors I_A, I_B and I_C may be resolved along Cartesian axes Ox and Oy, Ox being taken conveniently in the direction of I_A as shown in the sketch.

Then I_B resolves into two components, Ob_x and Ob_y, such that

$$Ob_y = 5{\cdot}8 \cos (110° - 90°),$$
$$= 5{\cdot}8 \cos 20°,$$
$$= 5{\cdot}45 \text{ amps.}$$
$$\text{and } Ob_x = 5{\cdot}8 \sin 20°,$$
$$= 1{\cdot}984 \text{ amps.}$$

No.	*Log.*
5·8	0·7634
cos 20°	$\bar{1}$·9730+
	0·7364
5·8	0·7634
sin 20°	$\bar{1}$·5341+
	0·2975

Similarly I_C may be resolved into the two components, Oc_x and Oc_y, where



$$Oc_y = 7{\cdot}4 \cos (115° - 90°),$$
$$= 7{\cdot}4 \cos 25°,$$
$$= 6{\cdot}707 \text{ amps,}$$
$$\text{and } Oc_x = 7{\cdot}4 \sin 25°,$$
$$= 3{\cdot}127 \text{ amps.}$$

No.	*Log.*
7·4	0·8692
cos 25°	$\bar{1}$·9573+
	0·8265
7·4	0·8692
sin 25°	$\bar{1}$·6259+
	0·4951

The components Ob_x and Oc_x act in the same direction, opposite to that of I_A, and are hence additive.

$$Ob_x + Oc_x = 1{\cdot}984 + 3{\cdot}127,$$
$$= 5{\cdot}111 \text{ amps.}$$

$\therefore$ The resultant vector in the direction Ox is given by

$$OR_x = 6{\cdot}2 - 5{\cdot}111,$$
$$= 1{\cdot}089 \text{ amps.}$$

The components Ob_y and Oc_y act in opposite directions along the axis Oy and hence their resultant is given by

$$Oc_y - Ob_y = 6{\cdot}707 - 5{\cdot}45,$$
$$= 1{\cdot}257 \text{ amps.}$$

and this will act in the direction Oc_y since this component is the greater. This is shown as OR_y in the sketch.

The resultant of OR_x and OR_y is given by

$$OR^2 = OR_x^2 + OR_y^2,$$
$$= 1{\cdot}089^2 + 1{\cdot}257^2,$$
$$= 1{\cdot}186 + 1{\cdot}580,$$
$$= 2{\cdot}766.$$
$$\therefore OR = 1{\cdot}663 \text{ amps.}$$
$$\text{Also } \tan \angle R_xOR = \frac{OR_y}{OR_x},$$
$$= \frac{1{\cdot}257}{1{\cdot}089}.$$
$$\therefore \angle R_xOR = 49° 6'.$$

No.	*Log.*
1·089	0·0370
$1{\cdot}089^2$	0·0740
1·257	0·0993
$1{\cdot}257^2$	0·1986

No.	*Log.*
1·257	0·0993
1·089	0·0370−
	0·0623

Hence, by calculation, $I_R = \underline{1{\cdot}663 \text{ Amps,}}$ and this $\underline{\text{lags } 49° 6'}$ behind I_A, using the conventional counterclockwise rotation of vectors.

Note.—The resultant I_R may be obtained graphically from the construction shown dotted in the sketch. In this, starting with OI_A and using the chain rule, the line I_AI_B is drawn parallel to OI_B and of appropriate magnitude to represent it to the same scale. Similarly I_BI_R is drawn parallel to OI_C and in the same direction, again using the same scale. Then the closing line OI_R represents the resultant I_R in magnitude and direction.

***Q.* 8.** *Derive from first principles an expression for* $\frac{dy}{dx}$, *where* $y = \frac{3}{x} + 4x + 5$.

Sketch the graph of y against x, showing both positive and negative values of x, and marking clearly any asymptotes.

Find the point (or points) on the curve at which $\frac{dy}{dx} = 0$.

A. 8. $y = \frac{3}{x} + 4x + 5.$ ..... (1)

Suppose x increases by a small amount δx and let δy be the corresponding change in y.

Then $y + \delta y = \frac{3}{x + \delta x} + 4(x + \delta x) + 5.$ ..... (2)

Subtracting equation (1) from equation (2) gives

$$\delta y = \frac{3}{x + \delta x} + 4x + 4\delta x + 5 - \frac{3}{x} - 4x - 5,$$

$$= \frac{3}{x + \delta x} - \frac{3}{x} + 4\delta x,$$

$$= \frac{3x - 3(x + \delta x)}{x(x + \delta x)} + 4\delta x,$$

$$= -\frac{3\delta x}{x(x + \delta x)} + 4\delta x.$$

Dividing both sides of the equation by δx gives

$$\frac{\delta y}{\delta x} = -\frac{3}{x(x + \delta x)} + 4.$$

In the limit as $\delta x \to 0$, $\frac{\delta y}{\delta x} = \frac{dy}{dx}$.

$$\therefore \frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x},$$

$$= \underline{-\frac{3}{x^2} + 4.}$$

The graph may be plotted from the following table of values.

x	-5	-4	-3	-2	-1	$-\frac{1}{2}$	$-\frac{1}{5}$	0
$\frac{3}{x}$	$-\frac{3}{5}$	$-\frac{3}{4}$	-1	$-1\frac{1}{2}$	-3	-6	-15	∞
$4x$	-20	-16	-12	-8	-4	-2	$-\frac{4}{5}$	0
5	5	5	5	5	5	5	5	5
$y = \frac{3}{x}+4x+5$	$-15\frac{3}{5}$	$-11\frac{3}{4}$	-8	$-4\frac{1}{2}$	-2	-3	$-10\frac{4}{5}$	∞

x	$\frac{1}{5}$	$\frac{1}{2}$	1	2	3	4	5
$\frac{3}{x}$	15	6	3	$1\frac{1}{2}$	1	$\frac{3}{4}$	$\frac{3}{5}$
$4x$	$\frac{4}{5}$	2	4	8	12	16	20
5	5	5	5	5	5	5	5
$y = \frac{3}{x} + 4x + 5$	$20\frac{4}{5}$	13	12	$14\frac{1}{2}$	18	$21\frac{3}{4}$	$25\frac{3}{5}$

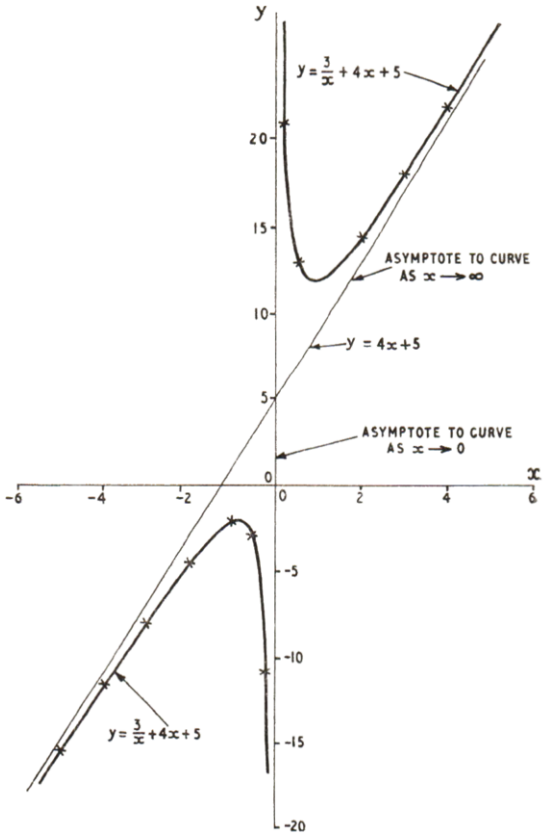
The graph is shown in the sketch, the values of y at $x = \pm\frac{1}{5}$ and $x = \pm\frac{1}{5}$ having been added in order to define the shape of the curve close to zero. As x approaches zero, either from a value below or from a value above zero, the term $\frac{3}{x}$ tends to infinity. In symbols this may be written as

$\lim_{x \nearrow 0} \frac{3}{x} = -\infty$ signifying x increasing from a negative value towards zero,

and $\lim_{x \searrow 0} \frac{3}{x} = +\infty$ signifying x decreasing from a positive value towards zero.

In either case the other terms of y, $4x$ and 5 become negligible as $x \to 0$ and hence $y \to \pm\infty$ as x approaches zero.
Hence, the y-axis is an asymptote to the curve at $x = 0$.

Also, as x becomes very large in magnitude, the term $\frac{3}{x}$ tends to zero and may be neglected in comparison with $4x + 5$. Hence, as



$x \to \infty$, $y \to 4x + 5$. Thus the line $y = 4x + 5$ shown in the sketch is an asymptote to the curve $y = \frac{3}{x} + 4x + 5$.

$$\frac{dy}{dx} = -\frac{3}{x^2} + 4,$$

$$\therefore \text{ when } \frac{dy}{dx} = 0, \text{ then } -\frac{3}{x^2} + 4 = 0,$$

$$\text{or, } \frac{3}{x^2} = 4.$$

$$\therefore x^2 = \tfrac{3}{4},$$

$$\text{or, } x = \frac{\pm\sqrt{3}}{2} = \pm 0{\cdot}866.$$

But this is the condition for maximum or minimum values and from the sketch it is evident that a maximum occurs at $x = -0{\cdot}866$ and a minimum at $x = 0{\cdot}866$.

When $x = -0{\cdot}866$,

$$y = \frac{3}{-\sqrt{3}/2} + 4\frac{-\sqrt{3}}{2} + 5,$$

$$= -2\sqrt{3} - 2\sqrt{3} + 5,$$
$$= -6{\cdot}928 + 5,$$
$$= -1{\cdot}928.$$

When $x = +0{\cdot}866$,

$$y = 6{\cdot}928 + 5,$$
$$= 11{\cdot}928.$$

These two points may also be plotted in order to define the curve more clearly.
Thus, the points on the curve at which $\frac{dy}{dx} = 0$ are,

$\underline{x = -0{\cdot}866, y = -1{\cdot}928, \text{ which is a maximum;}}$

and $\underline{x = +0{\cdot}866, y = 11{\cdot}928, \text{ which is a minimum.}}$

Q. **9.** (*a*) *Evaluate*

(*i*) $\int_1^4 \sqrt{x}\,dx,$

(*ii*) $\int_0^1 (3x - x^3)^2 dx$.

(*b*) *Find the mean value of the function* $y = 2x^2 - 5x + 2$ *from* $x = \frac{1}{2}$ *to* $x = 2$. *Sketch the graph of the function to show the area involved in this integration process.*

A. 9. (*a*) (*i*)
$$\int_1^4 \sqrt{x}\,dx = \int_1^4 x^{1/2} dx,$$
$$= \left[\frac{x^{3/2}}{3/2}\right]_1^4,$$
$$= \tfrac{2}{3}[4^{3/2} - 1^{3/2}],$$
$$= \tfrac{2}{3}[8 - 1],$$
$$= \underline{\frac{14}{3}}.$$

(*ii*)
$$\int_0^1 (3x - x^3)^2 dx = \int_0^1 (9x^2 - 6x^4 + x^6) dx,$$
$$= \left[\frac{9x^3}{3} - \frac{6x^5}{5} + \frac{x^7}{7}\right]_0^1,$$
$$= \left[3 - \frac{6}{5} + \frac{1}{7}\right] - [0],$$
$$= \underline{\frac{68}{35}}.$$

(*b*) $y = 2x^2 - 5x + 2$.

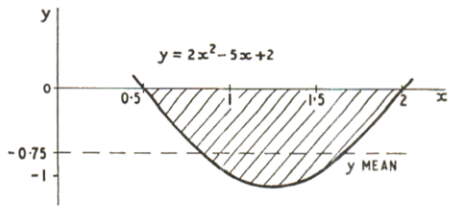
The mean value of y from $x = \frac{1}{2}$ to $x = 2$ is given by,

$$y_{mean} = \frac{\int_{\frac{1}{2}}^2 (2x^2 - 5x + 2) dx}{2 - \frac{1}{2}},$$
$$= \frac{\left[\frac{2x^3}{3} - \frac{5x^2}{2} + 2x\right]_{\frac{1}{2}}^2}{3/2},$$
$$= \frac{2}{3}\left[\left\{\frac{2}{3}2^3 - \frac{5}{2}2^2 + 4\right\} - \left\{\frac{2}{3}(\tfrac{1}{2})^3 - \frac{5}{2}(\tfrac{1}{2})^2 + 1\right\}\right],$$
$$= \frac{2}{3}\left[\frac{16}{3} - 10 + 4 - \frac{1}{12} + \frac{5}{8} - 1\right],$$
$$= \frac{2}{3}\left[\frac{128 - 2 + 15}{24} - 7\right],$$
$$= \frac{2}{3}\left[\frac{141 - 168}{24}\right],$$
$$= -\frac{2}{3} \times \frac{27}{24},$$
$$\therefore\ y_{mean} = \underline{-\frac{3}{4}}.$$

The graph may be sketched from the following values.

x	$\frac{1}{2}$	1	$1\frac{1}{2}$	2
$2x^2$	$\frac{1}{2}$	2	$4\frac{1}{2}$	8
$-5x$	$-2\frac{1}{2}$	-5	$-7\frac{1}{2}$	-10
2	2	2	2	2
$y = 2x^2 - 5x + 2$	0	-1	-1	0

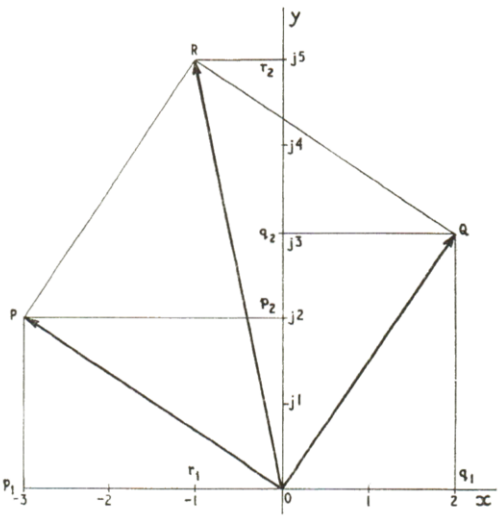
The graph is shown in the sketch which shows that, between $x = \frac{1}{2}$ and $x = 2$, the graph lies below the x-axis. The area between the curve and the x-axis is given by $\int_{\frac{1}{2}}^2 y\,dx$ and is wholly negative. The mean value obtained above is shown in the sketch and the area involved is shaded.



***Q.* 10.** (*a*) *On the same diagram draw the vectors which represent the complex numbers* $-3 + j2$ *and* $2 + j3$ *respectively. Prove from the diagram that these vectors are perpendicular.*
Show also the sum of the two vectors on the diagram.
(*b*) *Express in the form* $a + -jb$ *the roots of the equation*

$$5z^2 - 4z + 8 = 0.$$

A. 10. (*a*) In the Argand diagram of the sketch, $\vec{OP}$ is the vector representing the complex number $-3 + j2$ and $\vec{OQ}$ is the vector representing $2 + j3$.



From P draw perpendiculars Pp_1 and Pp_2 to the x and y-axes respectively; similarly, from Q, draw Qq_1 and Qq_2 perpendicular to the respective axes.
Then, in triangles OPp_1 and OQq_1

$\angle Pp_1O = \angle Qq_1O$ right angles,
$Op_1 = Qq_1$ numerically,
and $Pp_1 = Oq_1$ numerically.

Hence the triangles are congruent.

$$\therefore\ \angle POp_1 = \angle OQq_1. \qquad \ldots\ldots (1)$$

But, in triangle OQq_1,

$$\angle QOq_1 + \angle OQq_1 + 90° = 180°,$$
$$\text{or } \angle QOq_1 + \angle OQq_1 = 90°. \qquad \ldots\ldots (2)$$

Adding $\angle QOq_1$ to both sides of equation (1) gives

$$\angle POp_1 + \angle QOq_1 = \angle OQq_1 + \angle QOq_1,$$
$$= 90°. \text{ from equation (2)}$$
$$\text{But } \angle p_1Oq_1 = 180°,$$
$$\therefore\ \angle POQ = 180° - (\angle POp_1 + \angle QOq_1),$$
$$= 180° - 90°,$$
$$= \underline{90°}. \qquad \text{Q.E.D.}$$

If PR is drawn through P parallel to OQ and QR is drawn through Q parallel to OP so as to complete the rectangle OPRQ, then $\vec{OR}$ is the vector sum of $\vec{OP}$ and $\vec{OQ}$.
This may be proved by projecting $\vec{OR}$ onto the x-axis and y-axis to give Rr_1 and Rr_2 respectively.
It can be seen that:

$$\vec{Or_1} = \vec{Op_1} + \vec{p_1r_1},$$
$$= \vec{Op_1} + \vec{Oq_1},$$
$$= -3 + 2 = -1.$$
$$\text{and } \vec{Or_2} = \vec{Op_2} + \vec{p_2r_2},$$
$$= \vec{Op_2} + \vec{q_1Q},$$
$$= j2 + j3 = j5.$$
$$\text{Thus, } \vec{OR} = \vec{Or_1} + \vec{Or_2},$$
$$= -1 + j5.$$

which is the sum of $-3 + j2$ and $2 + j3$.

(*b*) $5z^2 - 4z + 8 = 0.$

$$\therefore\ z = \frac{4 \pm \sqrt{16 - 4 \times 5 \times 8}}{10},$$

$$= \frac{4 \pm 4\sqrt{1 - 10}}{10},$$

$$= \frac{2}{5} \pm \frac{2}{5}\sqrt{-9},$$

$$= \frac{2}{5} \pm j\frac{6}{5}.$$

Thus the roots of $5z^2 - 4z + 8 = 0$ are

$$\underline{\frac{2}{5} + j\frac{6}{5} \text{ and } \frac{2}{5} - j\frac{6}{5}}.$$

TELECOMMUNICATION PRINCIPLES B, 1968

Students were expected to answer three questions from Q. 1–4 and three from Q. 5–10.

Q. 1. *Explain the meaning of reactance.*

Plot the reactance-frequency characteristics over the range 0 to 400 Hz (c/s) for,

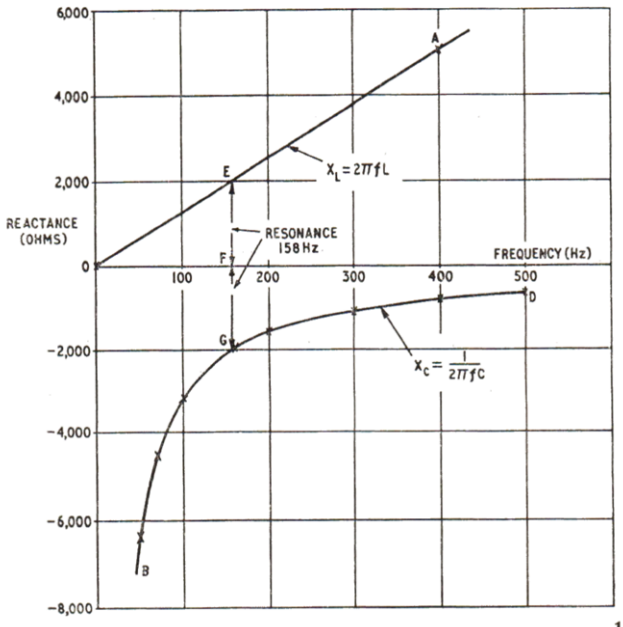
(*a*) *an inductor of 2H,*
(*b*) *a capacitor of 0·5 μF.*

Show how the frequency of resonance for a circuit containing 2H in series with 0·5 μF can be deduced from these curves.

Calculate this frequency. If the effective resistance of the series circuit is 10 ohms calculate the Q-value at resonance.

A. 1. Reactance is the ability of a circuit to resist the passage of an alternating current without the dissipation of energy. It is given by the ratio of the component of the alternating voltage across the terminals of a network in quadrature with the current flowing, divided by the current.

(*a*) The reactance of an inductor L H at a frequency f Hz is given by $2\pi fL$ ohms. The reactance of a given inductor is therefore proportional to the frequency of the alternating current passing through it and if inductive reactance is plotted against frequency a straight line results. To plot this line only two points are required. When the frequency is zero, the reactance is zero and when the frequency is 400 Hz the reactance of a 2 H inductor is $2\pi 400 \times 2$, i.e. 5,027 ohms. The straight line is OA in the sketch.



(*b*) The reactance of a capacitor C farads at frequency f Hz is $\frac{1}{2\pi fC}$ and this is negative in relation to inductive reactance. The curve is a rectangular hyperbola. It is necessary to calculate a few points on this curve to obtain the graph of the reactance, shown in the sketch as BD. Since the reactance of a capacitor is negative the curve is in the fourth quadrant.

f (Hz)	50	100	200	300	400	500
X, (ohms)	6,360	3,180	1,590	1,060	795	636

Resonance of a series L–C circuit occurs when the overall reactance is zero, i.e. the inductive and capacitive reactances are equal. This condition can be obtained by inspection from the two curves of the sketch as shown at EFG.

The resonance frequency is given by $f = \frac{1}{2\pi\sqrt{LC}}$

When $L = 2$ H and $C = 0{\cdot}5 \times 10^{-6}$ farads.

$$f = \frac{1}{2\pi\sqrt{2 \times 0{\cdot}5 \times 10^{-6}}} = \frac{10^3}{2\pi} = \underline{158} \text{ Hz}.$$

The Q factor is the ratio of the reactance of the inductor (or capacitor) at the frequency of resonance to the circuit resistance, i.e.

$$Q = \frac{\omega L}{R}.$$

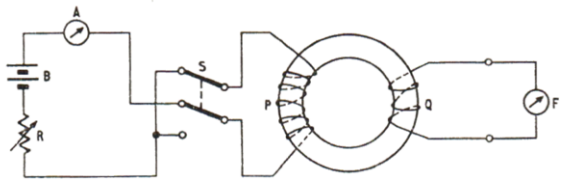
$$\therefore\ Q = \frac{2\pi \times 158 \times 2}{10} = \underline{198{\cdot}4}.$$

Q. 2. *Describe an experiment to determine the magnetization (B–H) curve for a ring-shaped specimen of iron. Sketch a curve to show a typical result.*

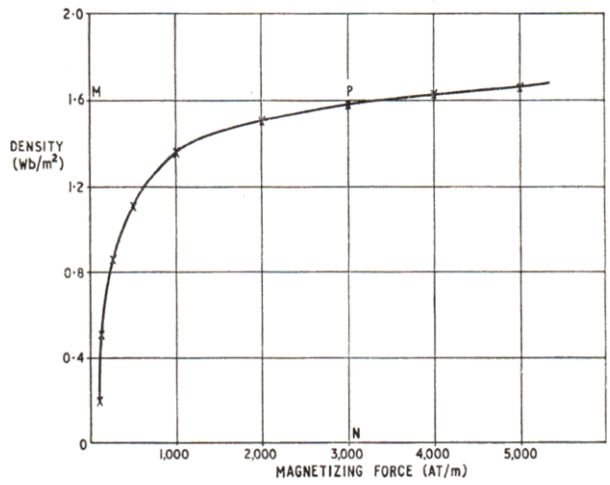
Define relative permeability and show how it can be deduced from the results of your experiment.

What deductions on the hysteresis loss in the iron can be made from the B–H loop?

A. 2. A circuit for the experimental determination of the magnetization curve of a specimen of iron in the shape of a ring, is shown in sketch (*a*).



(*a*)



(*b*)

Two coils are wound on the ring. The primary coil, P, is used to generate a magnetic flux in the iron, and the secondary coil, Q, produces an induced e.m.f. that indicates the quantity of flux. Current is measured by ammeter A, supplied by battery B, and can be adjusted by resistor R and reversed in the coil P by reversing switch S. If coil P has N_p turns in which the current is I, the magneto-motive-force (m.m.f.) is N_pI ampere turns.

The secondary coil Q of N_q turns is connected to a flux-meter F. This is a ballistic galvanometer giving a deflexion proportional to the amount of energy put into it, which is proportional to the total flux change in Q. When switch S is thrown to reverse the primary current, the deflexion of F is proportional to the magnetic flux change in the iron while the current in the primary is changing.

A series of readings of F for known values of primary current I as measured by ammeter A is then taken.

The reversing switch S must be thrown several times with successively decreasing current before each reading on the flux meter is taken to restore the iron to a neutral magnetic condition.

A typical B–H curve resulting from this experiment is shown in sketch (*b*).

The relative permeability of an iron core is the ratio of the flux density produced in that core to the flux density produced in air by the same magnetizing force when the core is removed.

For the M.K.S. system of units, if B is the flux density and H the magnetizing force, for a non-magnetic material, e.g. air,

$$B = \mu_0 H,$$

where μ_0 is the magnetic permeability of free space.

If an iron core of relative permeability μ_r is introduced then:

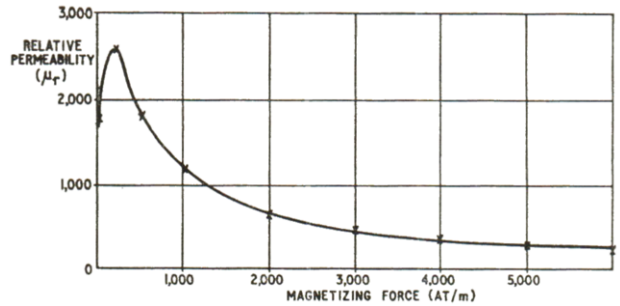
$$\frac{B}{H} = \mu_r\mu_0 = \text{absolute permeability},$$

$$\text{and, } \mu_r = \frac{1}{\mu_0}\frac{B}{H}.$$

As $\mu_0 H$ is equal to the ampere turns per metre in the coil,

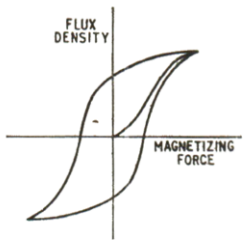
$$\mu_r = \frac{\text{flux density}}{\text{ampere turns per metre}}.$$

The relative permeability at a point P on the curve of sketch (*b*) is therefore the ratio of PM, the flux density, to PN, the ampere turns per metre producing the flux. This ratio is plotted against the ampere turns per metre in sketch (*c*).



(*c*)

A typical hysteresis loop for iron is shown in sketch (*d*). The area of this loop is proportional to the energy dissipated in the iron per cycle of alternating flux.



(*d*)

A wide loop, as in sketch (*d*), denotes a sample of iron having a fairly high hysteresis loss. A low-loss iron would give a narrow B–H loop having a small area.

Q. **3.** *What are the essential features of a transformer?*
Why is it usual to laminate the core of a low-frequency transformer?
A transformer operating from a 240*-volt,* 50 *Hz* (*c/s*) *supply is to be used to provide* 80 *volts across a* 120*-ohm load resistance. Assuming that there are no losses in the transformer, calculate*

(*a*) *the turns ratio,*
(*b*) *the currents in the primary and secondary circuits,*
(*c*) *the equivalent load resistance on the supply.*

What is the phase relation between the currents in the primary and secondary circuits?

A. 3. A transformer consists essentially of two insulated coils, the primary and secondary, mounted in close proximity so that there is a substantial degree of magnetic coupling between them. It is usual for the two coils to have a common iron core if the transformer is for operation at power, or audio, frequencies. This core should be a complete ring in order that all the magnetic flux is constrained to remain within this low-reluctance magnetic circuit.

When an alternating current is passed through the primary coil of N_1 turns, an alternating flux is set up sympathetically in the iron core. The second coil, being wound on the same core, is cut by this flux. An alternating e.m.f., proportional to the number of turns N_2 on the secondary, is then induced in this secondary coil.

Provided that all the flux from one coil cuts the other, the r.m.s. e.m.f. E_2 across the secondary is given by

$$E_2 = \frac{N_2}{N_1} \times E_1,$$

where E_1 is the r.m.s. voltage applied to the primary.

Because the magnetic flux is continuously changing in a transformer core, internal e.m.f.'s will be induced in the body of the core itself. These, if unimpeded, will lead to heavy I^2r losses within the iron. The ohmic path resistance to these eddy currents can be made very high by making the core out of many layers of thin (e.g. 0·020 in) iron, each insulated from its neighbour by a thin layer of paint or tissue paper. The iron laminations are continuous in the direction of the magnetic flux so the magnetic reluctance is not increased by using laminations. The e.m.f. induced within the core will be at right angles to the flux, i.e. across the laminations, and no currents can flow in this direction because of the insulating layers between laminations. Hence, eddy current loss is minimal as a result of the laminated structure of the core.

(*a*) In the problem $E_1 = 240$ volts,
$E_2 = 80$ volts.

$$\therefore \text{ Turns Ratio} = \frac{E_1}{E_2} = 3 : 1.$$

(*b*) The secondary coil gives 80 volts across the 120-ohm load.

$$\therefore I_2 = \frac{2}{3} \text{ amps.}$$

As the transformer is ideal there are to be no energy losses.

$$\therefore \text{ output power} = \text{input power},$$
$$80 \times (\tfrac{2}{3}) = 240\, I_1.$$
$$\therefore I_1 = \frac{2}{9} \text{ amps.}$$

(*c*) Input power $= E_1I_1 = 240 \times \frac{2}{9} = \frac{480}{9}$ watts. ..... (1)

Let the equivalent load resistance on the supply be R ohms,

Then the power into this load $= I_1^2R = \left(\frac{2}{9}\right)^2 R.$ ..... (2)

Equating equations (1) and (2) gives

$$\left(\frac{2}{9}\right)^2 R = \frac{480}{9},$$

$$\therefore R = \frac{480}{9} \times \left(\frac{9}{2}\right)^2 = 120 \times 9.$$

$$\therefore R = \underline{1{,}080 \text{ ohms.}}$$

An alternative approach is as follows:

$$\text{Equivalent load resistance} = \left(\frac{N_2}{N_1}\right)^2 \times R_2,$$
$$= 3^2 \times 120,$$
$$= \underline{1{,}080 \text{ ohms.}}$$

The currents in the primary and secondary in an ideal transformer on resistance load are in anti-phase, i.e. there is a phase difference of 180°. This follows from Lenz's Law, that an induced e.m.f. must be in a direction that tends to oppose the change creating the flux-linkage, in this case the primary current.

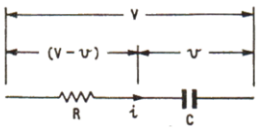
Q. 4. *When a capacitor (C) in series with a resistor (R) is connected to a constant voltage source (V) the p.d. across the capacitor rises slowly. Explain the reason for this.*
Give expressions for

(*a*) *the time constant of the circuit,*
(*b*) *the p.d. across the capacitor,*
(*c*) *the charging current,*

in relation to the circuit constants and time from switching-on the voltage.
A circuit consisting of 50 μF in series with a resistor is to have a time constant of 2 seconds. What value of resistor will be needed?
If the circuit is connected to a 100-volt d.c. supply, calculate the charging current

(*i*) *initially,*
(*ii*) *2 seconds after switching on.*

A. 4. In the circuit (sketch (*a*)), by Ohm's Law,



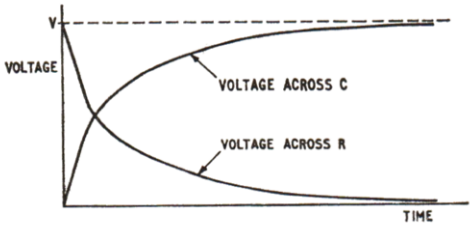
(*a*)

$$i = \frac{\text{voltage across } R}{R},$$

$$= \frac{V - v}{R}. \qquad \ldots\ldots (1)$$

As the capacitor charges, v rises; hence the p.d. across the resistor must fall, so that the current, i, gradually decreases.
The graph of voltage, v, against time, t, is an exponential curve sketch (*b*), given by:

$$v = V\left[1 - e^{-\frac{t}{CR}}\right].$$



(*b*)

The charging current is given by expression (1).

$$\therefore \text{ Charging current} = \frac{V - V\left[1 - e^{-\frac{t}{CR}}\right]}{R},$$

$$= \frac{V}{R} e^{-\frac{t}{CR}}.$$

The product CR is known as the time constant, given as 2 seconds.

$$\text{Since } C = 50 \times 10^{-6} \text{ farads,}$$

$$R = \frac{2}{50 \times 10^{-6}},$$

$$= 40{,}000 \text{ ohms.}$$

The initial charging current [$t = 0$] is due to the full 100 volts across the 40,000-ohm resistor since there is initially no p.d. across C.

$$\therefore i_0 = \frac{100}{40{,}000} = \underline{2{\cdot}5 \text{ mA.}}$$

After 2 seconds, i.e. the time constant, $t = CR$.

$$\therefore \frac{t}{CR} = 1.$$

$$\therefore \text{ The current after 2 seconds} = \frac{V}{R} e^{-1} = \frac{V}{R}\frac{1}{e}$$

$$= \frac{I_0}{e} = \frac{2{\cdot}5 \times 10^{-3} \text{mA}}{2{\cdot}718},$$

$$= \underline{0{\cdot}921 \text{ mA.}}$$

Q. 5. *The coil of an electromagnetic relay has an inductance of 5H when the relay armature is not operated. It increases to 20H when the armature is operated. Explain why there is an increase in inductance. The relay coil-circuit has a d.c. resistance of 240 ohms and is energized from a 50-volt battery supply. The armature operates with 20 mA in the coil. Sketch a curve showing how the current rises.*

Give expressions for the current/time relations for the two portions of the curve.
What are the two time constants?

A. 5. The self-inductance of a coil is due to the back e.m.f. induced in it by changes in the magnetic flux created by current in the coil itself. The self-inductance [usually called the 'inductance'] is given by the product of the number of turns on the coil and the flux per ampere. Hence, the larger the flux per ampere, the larger the inductance will be.
In an electro-magnetic relay, the number of turns is constant. When the armature is not operated there is a large gap in the magnetic circuit. The magnetic reluctance is therefore high and the flux generated (by unit current in the coil) through the magnetic core is low. The relation is given by:

$$\text{flux} = \frac{\text{magneto-motive-force}}{\text{reluctance}}.$$

When the armature gap is closed, the reluctance of the magnetic circuit through the coil core and armature is much lower. Therefore, the flux per unit current is greater. Hence, the inductance has increased by operating the relay armature.
The current, i, at a time, t, seconds from switch-on is given by:

$$i = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right).$$

For the first stage with the relay non-operated, the inductance is 5 H.

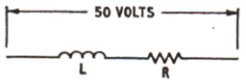
$$\therefore \text{ coil current at time } t = \frac{50}{240}\left(1 - e^{-\frac{240}{5}t}\right),$$

$$\text{or } i = \underline{\frac{5}{24}(1 - e^{-48t}).}$$

For the second stage after the relay has operated, the inductance becomes 20 H.

$$\therefore i = \frac{50}{240}\left(1 - e^{-\frac{240}{20}t}\right),$$

$$\text{or } i = \underline{\frac{5}{24}(1 - e^{-12t}).}$$



(*a*)

Time constant for an inductance L in series with a resistance R (sketch (*a*)) $= \frac{L}{R}$ seconds,

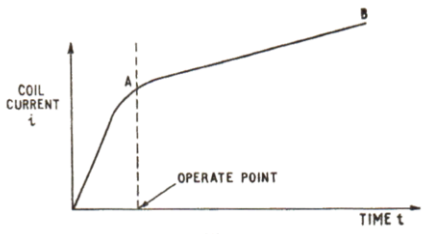
Time constant for the 5 H inductor in series with 240 ohms

$$= \frac{5}{240} = \underline{20{\cdot}8 \text{ms.}}$$

Time constant for 20 H in series with 240 ohms

$$= \frac{20}{240} = \underline{83{\cdot}3 \text{ms.}}$$

The current-time curve consists of an exponential portion, OA, of 20·8 ms time constant, followed, by an exponential portion, AB, having less steep rate of rise, with time constant 83·3 ms. This curve is shown in sketch (*b*).



(*b*)

Q. **6.** *By considering a transistor as two junction diodes, or otherwise, explain the principle of a transistor as an amplifying device.*

Give a circuit diagram of a single-stage transistor amplifier connected in common emitter configuration and with load resistor.

Sketch typical collector-current/collector-voltage characteristics for this amplifier.

Show by means of a load line how the gain can be determined.

Q. **7.** *Determine by means of phasors (vectors) drawn to scale, or otherwise, the magnitude of the current flowing and its phase relative to the supply voltage when each of the circuits shown is connected separately across a 100-volt 100-kHz (kc/s) supply.*

Find the total current taken and its phase relative to the supply voltage when the two circuits are connected in parallel.

Fig. 1

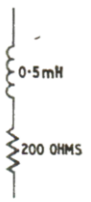
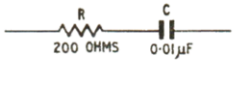


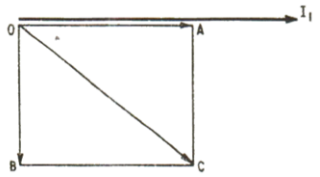
Fig. 2

A. 7. In the series circuit (sketch (*a*)) the current I_1 will be the same in the resistor and the capacitor. Let OI_1 be the direction of the current phasor (sketch (*b*)). Then 200 I_1 will be the voltage across the 2,000 ohm resistor, in phase with I_1.

Let OA represent 200 I_1.



(*a*)



(*b*)

The reactance of 0·01 μF at 100 kHz $= \dfrac{1}{2\pi fC}$,

$= \dfrac{10^6}{2\pi 10^5 \times 10^{-2}}$,

$= 159{\cdot}5$ ohms.

Since current leads voltage by 90° in a capacitor, OB represents the voltage across capacitor, where OB represents 159·5 I_1.

Therefore OC, the diagonal of rectangle OACB, represents the overall voltage.

Now $OC^2 = OA^2 + OB^2$,

$= (200^2 + 159^2)\, I_1^2$.

$\therefore$ OC $= 256\, I_1$.

But, OC = 100 volts r.m.s.

$$\therefore\ I_1 = \frac{100}{256} = \underline{391 \text{ mA}}.$$

The phase angle between I_1 and the applied voltage $= \angle$AOC,

$$= \tan^{-1}\frac{159{\cdot}5}{200} = \underline{38°36'}.$$

Now consider the series circuit of sketch (*c*),

(*c*)

reactance of $L = 2\pi fL$,

$= 2\pi 10^5 \times 0{\cdot}5 \times 10^{-3}$,

$= 100\pi$,

$= 314$ ohms.

Let I_2 be the current in the inductor and series resistor.

Let DI, represent the direction of the current phasor in sketch (*d*). Then DF represents the voltage across the series resistor, where DF represents 200 I_2.

DH represents the inductive voltage, leading the current I_2 by 90°. If DG is the resultant voltage across the circuit, by Pythagoras Theorem

$DG^2 = I_2^2[314^2 + 200^2]$.

$\therefore$ DG $= 373\, I_2$.

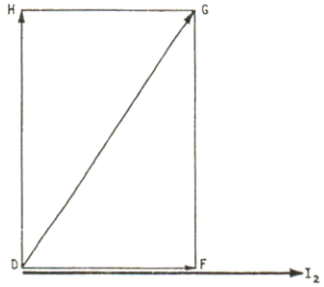
But DG represents 100 volts.

$$\therefore\ I_2 = \frac{100}{373} = \underline{268 \text{ mA}}.$$

Phase Angle = DGF,

$$= \tan^{-1}\frac{314}{200} = \tan^{-1} 1{\cdot}57,$$

$$= \underline{57°30'}.$$



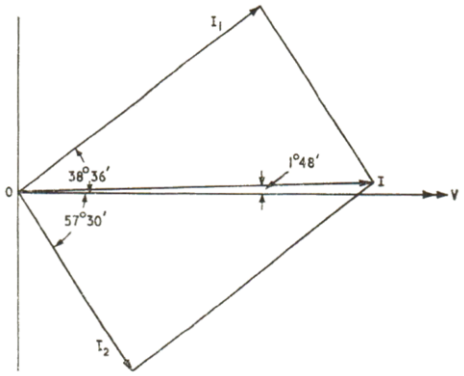
(*d*)

When the two circuits are in parallel and connected across a 100-volt 50-Hz supply, the total resultant current will be the vector sum of the currents in the two arms.

i.e. $I_1 = 0{\cdot}391$ amp making a phase angle of +38° 36′ to the voltage,

and $I_2 = 0{\cdot}268$ amp making a phase angle of +57° 30′ to the voltage.

The resultant total current I, can be readily obtained by means of a phasor diagram as shown in sketch (*e*) or by calculation as follows.



(*e*)

Resolving horizontal components,

$I_{1\,\text{HOR}} + I_{2\,\text{HOR}} = 0{\cdot}391\ (\cos 38° 36') + 0{\cdot}268\ (\cos 57° 30')$,

$= 0{\cdot}3056 + 0{\cdot}1440 = 0{\cdot}4496$.

Resolving vertical components,

$I_{1\,\text{VER}} - I_{2\,\text{VER}} = 0{\cdot}391\ (\sin 38° 36'), - 0{\cdot}268\ (\sin 57° 30')$,

$= 0{\cdot}2440 - 0{\cdot}2260 = 0{\cdot}018$.

$\therefore$ Total current $I = \sqrt{0{\cdot}4496^2 + 0{\cdot}018^2}$,

$= \underline{0{\cdot}45 \text{ amps}}$.

Phase angle $\phi = \tan^{-1}\dfrac{0{\cdot}018}{0{\cdot}4496}$,

$= \underline{1° 48'}$

$\therefore$ Total current = 450 mA at a phase angle of 1° 48′ to the voltage.

Q. **8.** *Describe the principle of the d.c. generator.*

How is the air-gap flux normally produced in such a machine?

Explain briefly how the output voltage is related to the speed of rotation of the armature when the generator is connected to a constant-resistance load.

Give reasons for the shape of the voltage-speed curve.

A. 8. In the small d.c. generator, the air-gap flux is produced by either permanent magnets or shunt-connected field coils on iron cores terminating in pole faces. For larger generators, shunt-connected field coils are normal. A separate steady-voltage source of d.c. may sometimes be used for the exciting current when a constant flux is essential, or if a large degree of control over the air-gap flux is required.

The e.m.f. generated = average rate of change of flux linkage,
= number of armature conductors × length of conductor × air-gap flux-density × flux reversals per second.

The number of armature conductors and the length of these conductors is constant and the number of flux reversals is proportional to the speed of rotation.

If the air-gap flux is constant, as with a permanent-magnet field, or constant voltage external excitation, then: e.m.f. generated is proportional to speed of rotation.

The armature resistance must be allowed for when current is flowing into a load. If the load is a constant resistance, the load current will rise when the e.m.f. rises; the internal voltage drop in the armature will then also rise, and this voltage drop must be subtracted from the armature e.m.f. to obtain the terminal voltage. Hence, even with a constant air-gap flux the terminal voltage on load will not rise in proportion to the speed of rotation but will drop slightly below this proportionality. In practice, this voltage drop may be increased by the effect of "armature reaction" which will tend to weaken the effective air-gap flux.

***Q.* 9.** *The anode-current anode-voltage characteristics of a triode valve are given in the table. Describe briefly an experiment in which these results could be obtained.*

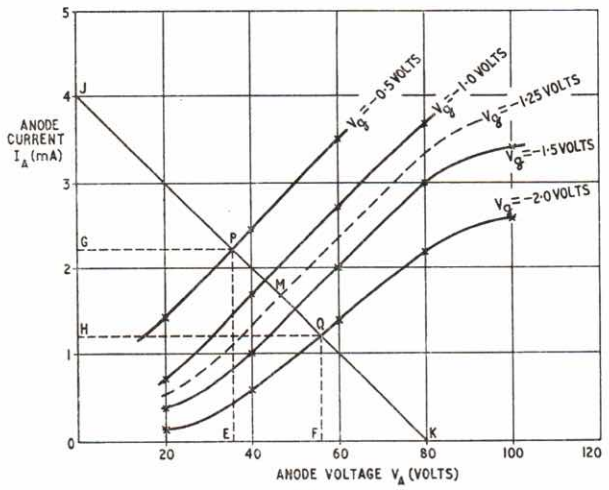
V_a	20	40	60	80	100
I_a *with* $V_g = -0{\cdot}5$	1·3	2·45	3·5	—	—
I_a *with* $V_g = -1{\cdot}0$	0·7	1·70	2·7	3·7	—
I_a *with* $V_g = -1{\cdot}5$	0·4	1·0	2·0	3·0	3·4
I_a *with* $V_g = -2{\cdot}0$	0·15	0·6	1·4	2·2	2·6

I_a *in milliamps,* V *in volts.*

The valve is to be used as an amplifier with a 20,000*-ohm anode load and an h.t. supply of* 80 *volts. Plot the characteristic curves and draw the load line, explaining your construction.*

Find the r.m.s. output voltage and a.c. output power for an input signal of 1·5 *volts peak-to-peak between grid and cathode, the grid bias voltage being* −1·25 *volts.*

A. 9. The anode current (I_A)–anode voltage (V_A) curves are shown in sketch (*a*). Each curve represents the I_A–V_A relation for a constant



(*a*)

grid-bias voltage. The curve for $V_g = -1{\cdot}25$ volts is drawn by interpolation between adjacent curves. M is the quiescent operating point, where the load line cuts the I_A–V_A curve for $V_g = -1{\cdot}25$ volts.

The load line is drawn with a gradient equal to the inverse of the load resistance. The load line can be constructed as follows:

When the anode current is zero, the anode voltage equals the battery voltage, which is 80 volts.

This fixes point K on the axis.

When the anode current is 4 mA, the whole anode voltage is across the 20,000-ohm load, since the h.t. supply voltage is only 80 volts. The anode p.d. will then be zero. This fixes J at 4 mA, for a value of V_A that is zero.

The load line is the straight line joining JK.

$$\text{Gradient of JK} = \frac{4 \cdot 10^{-3}}{80},$$

$$= \frac{1}{20{,}000}, \text{ i.e. the inverse of the load resistance.}$$

For a 20,000-ohm load, any grid input-voltage will produce an anode load condition that falls on this load line.

M is the quiescent point.

P and Q are the operating points for inputs of −0·5 volts r.m.s. and −0·2 volts r.m.s., respectively.

The anode swing is 1·5 volts between P and Q along the load line. The output voltage is EF, the projection of PQ on the axis.

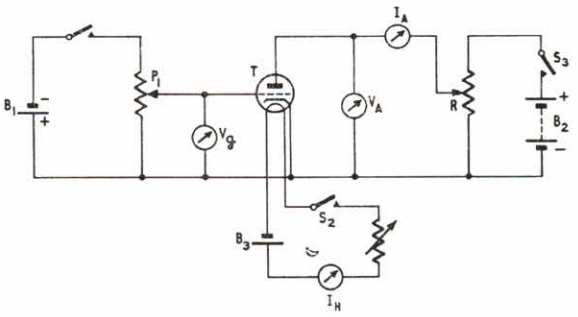
$$\text{EF} = 56 - 36 = 20 \text{ volts peak-to-peak,}$$

$$\text{and output voltage} = \frac{20}{2\sqrt{2}} \text{volts r.m.s.}$$

The output current is GH.

$$\text{GH} = (2{\cdot}15 - 1{\cdot}2),$$
$$= 0{\cdot}95 \text{ mA peak-to-peak,}$$
$$\text{and output current} = \frac{0{\cdot}95}{2\sqrt{2}} \text{mA r.m.s.}$$
$$\text{The output power} = \text{r.m.s. current} \times \text{r.m.s. voltage,}$$
$$= \frac{20}{2\sqrt{2}} \times \frac{0{\cdot}95}{2\sqrt{2}},$$
$$= \frac{19}{8},$$
$$= \underline{2{\cdot}38 \text{ mW}}.$$
$$\text{Output volts} = \frac{20}{2\sqrt{2}} \text{volts,}$$
$$= \underline{7{\cdot}0 \text{ volts r.m.s.}}$$

A test circuit for the triode T is shown in sketch (*b*). Battery B1, positive to earth, gives grid bias adjustable at P1. V_g is a voltmeter to measure grid voltage. Battery B2, negative to earth, supplies anode



(*b*)

current I_a with an anode voltage V_a. The load resistor R can be adjusted to give a range of anode voltage values. The heater current I_H should be kept constant at the recommended value for the valve.

The grid voltage is adjusted to a suitable value and resistor R adjusted to give the required range of anode voltage V_a. For each value of anode voltage the anode current I_A is noted. The grid voltage is adjusted to a new value and the procedure repeated.

Curves of anode current against anode voltage are plotted for each value of grid voltage. A family of curves similar to that of the sketch is obtained.

***Q.* 10.** *Why is the input impedance of a valve voltmeter high?*

How can a valve voltmeter be used to measure r.f. current?

A valve voltmeter is used with the circuit of Fig. 3 *to find the effective resistance* (r) *of an inductor of* 0·001H *at* 100 *kHz* (*kc/s*).

The valve voltmeter reads 5 *volts when connected across AB and* 43·9 *volts when across BC.*

Find the values of:

(*a*) *the* 100-*kHz current in the circuit,*
(*b*) *the effective series resistance* (*r*) *of the inductor.*

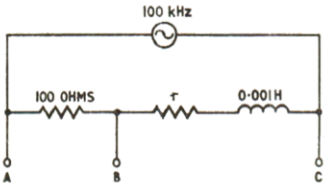


Fig. 3

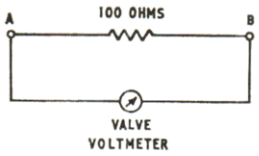
A. 10. The input impedance of a valve voltmeter can be made high, even at radio frequencies, because the meter input circuit is usually the grid of a triode valve, or a high-resistance diode rectifier-circuit followed by a d.c. amplifier stage. Such a high impedance causes negligible disturbance to the conditions in the circuit under test.

The triode grid operates on the electron stream of the triode by virtue of its electrostatic potential relative to the cathode. The grid does not draw current from the source of measurement except in so far as it has stray capacitance in shunt. This is very small, of the order of a few picofarads. Hence, it offers a high impedance at the input.

A diode input circuit uses the diode as a rectifier to charge a capacitor through a resistor, the p.d. across the capacitor representing the peak value of the voltage being measured. This rectified voltage is then passed to the grid of a d.c. amplifier valve for indication on an anode circuit ammeter. The diode circuit can be made high or low impedance as desired because the input impedance depends on the charging circuit.

A valve voltmeter will measure r.f. current if it is made to indicate the voltage across a resistor carrying this current. From Ohm's Law, the current is the measured voltage divided by the resistance across which it is measured. It is essential that stray shunt capacitance is kept very small to avoid a low shunt reactance at r.f. Connecting leads must be short; preferably the "diode probe" should be at the end, against the voltage being measured, to minimize the length of connecting leads carrying r.f. current.

As AB, sketch (*a*), is a 1,000-ohm resistor, the voltage across AB is in phase with the current in AB and is independent of frequency because there is no reactance component in the resistor.



(*a*)

Hence, by Ohm's Law, $i = \frac{V}{r}$,

$$= \frac{5}{100} = \underline{50 \text{ mA.}}$$

This is the current in the inductor.

At 100 kHz reactance of 0·001 H $= 2\pi\ 10^5\ 10^{-3}$,

$$= 628{\cdot}4 \text{ ohms.}$$

The voltage across this reactance carrying 0·05 amps = 31·420 volts and leads the current by 90°.

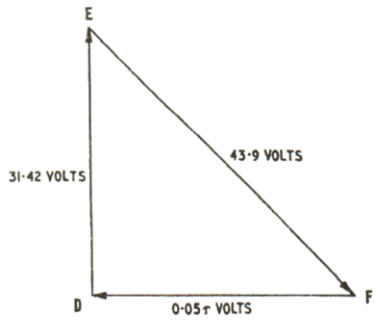
The voltage drop in the resistive component of r ohms = $0{\cdot}05r$ in phase with the current.

The vector triangle representing the resistance and inductance is DEF, where:

DE = 31·42, representing the voltage across the inductor,

FD = $0{\cdot}05i$ representing the voltage across the resistor r, and

EF = the voltage across BC in sketch (*b*).



(*b*)

Hence, $(0{\cdot}05r)^2 = (43{\cdot}9)^2 - (31{\cdot}42)^2 = 12{\cdot}48 \times 75{\cdot}32$,

$$= 940, = (30{\cdot}7)^2,$$

$$\therefore\ r = 30{\cdot}7 \times 20,$$

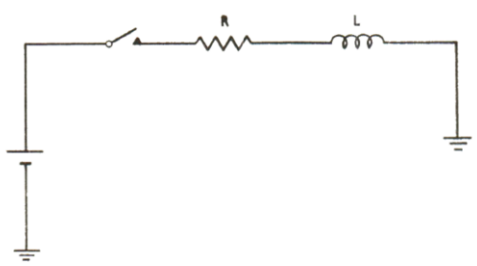
$$= \underline{614 \text{ ohms.}}$$

TELEGRAPHY B, 1968

Students were expected to answer any six questions.

***Q.* 1.** *A* 5-*H inductor of negligible resistance is joined in series with a resistor R and a battery. Draw approximate graphs showing the change of current with time if R has the values* (*a*) 1,000 *ohms* (*b*) 2,000 *ohms* (*c*) 4,000 *ohms. Assume that the voltage of the battery is adjusted so that the final current value is* 20 *mA in each case.*
Comment on the application of such graphs to telegraphy.

A. 1. The circuit is shown in the sketch (*a*).



(*a*)

For a circuit of this type, the current will rise to 0·6321 of its final value after a time, $t = \frac{L}{R}$,

where t is measured in seconds,
L is the inductance in henries, and
R is the resistance in ohms.

This value of t is known as the time-constant, and, for the values given,

$$(a)\ t = \frac{L}{R} = \frac{5}{1{,}000} = 5 \text{ ms},$$

$$(b)\ t = \frac{5}{2{,}000} = 2{\cdot}5 \text{ ms},$$

$$(c)\ t = \frac{5}{4{,}000} = 1{\cdot}25 \text{ ms}.$$

The current in each case will rise to 0·6321 of its final value,

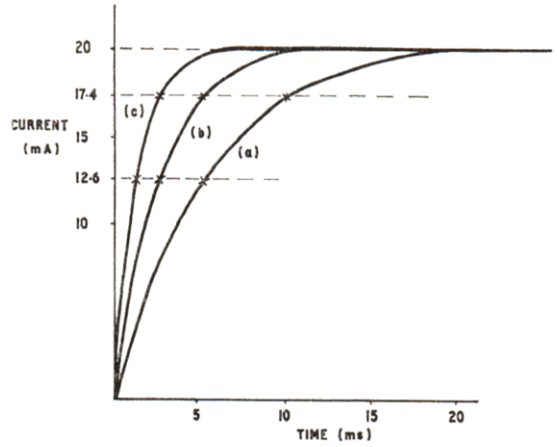
i.e. $0{\cdot}6321 \times 20$ mA = 12·642 mA.

Again, after periods of 5 ms, 2·5 ms and 1·25 ms respectively, the current will further increase by

$$0{\cdot}6321 \times (20 - 12{\cdot}6) = 4{\cdot}77 \text{ mA},$$

i.e. it will rise to $12{\cdot}64 + 4{\cdot}77 = 17{\cdot}4$ mA.

The current will reach the maximum value at a time equivalent to about four times the value of the time-constant, i.e. at about (*a*) 20 ms, (*b*) 10 ms, (*c*) 5 ms.



(*b*)

Using these values, the graphs may be plotted as shown in sketch (*b*). From the graphs, it can be seen that, as the resistance is increased, the maximum current through the inductance is reached in a shorter time. A similar set of curves could be drawn in reverse, showing the decay of current in the circuit. If a series of similar signals follow each other in rapid succession, the current in the inductance (which can represent a telegraph relay) may not reach the maximum value and may not fall to zero between signal elements. Under such conditions the receiver could fail to respond to the signals and so indicate incorrect characters. Thus, to give the best results, the resistance should be as great as possible, permitting an increased rate of signalling and more accurate reception.

It should be noted, however, that the steady value of the current will be influenced by the resistance, and for a given current, say 20 mA, the voltage applied to the transmit contacts must increase as the resistance is increased. In the example, the applied voltages to give a steady current of 20 mA will be, by Ohm's law, (*a*) 20 volts, (*b*) 40 volts, (*c*) 80 volts.

Q. 2. *Start-stop signals at 50 bauds are fed from two five-unit automatic transmitters into a synchronous multiplex system. In the synchronous system the start and stop elements are suppressed and the two five-unit input signals are interleaved by time-division.*

Calculate the aggregate modulation rate in bauds in the synchronous system; assume that the synchronous system is operating at exactly the speed of the combined input signals.

If both the five-unit inputs were converted into six-unit code before application to the synchronous system, what would then be the aggregate modulation rate?

A. 2. Each character of a start-stop signal consists of five character-elements of equal length, one start-element of length equal to one character-element, and a stop-element equivalent to one-and-a-half character-elements. A 50-baud signal has an element length of $\frac{1,000}{50} = 20$ milliseconds. Hence, one character has a length = 150 ms, with the start element = 20 ms and the stop element = 30 ms.

If two transmitters are used to transmit characters to a synchronous multiplex system, and the start and stop elements are suppressed, then 10 character elements are received and transmitted by the multiplex system every 150 ms, i.e. one element every 15 ms.

The aggregate modulation rate is then $\frac{1,000}{15} = \underline{66\frac{2}{3} \text{ bauds}}$.

If the 5-unit signals are converted to 6-unit signals, 12 character-elements are transmitted every 150 ms, i.e. one element every $\frac{150}{12} = 12\frac{1}{2}$ ms.

The aggregate modulation rate is then $\frac{1,000}{12\frac{1}{2}} = \underline{80 \text{ bauds}}$.

Q. 3. *Draw circuit diagrams for direct-current teleprinter line circuits which include a telegraph repeating-relay, for the two cases in which the line consists of*

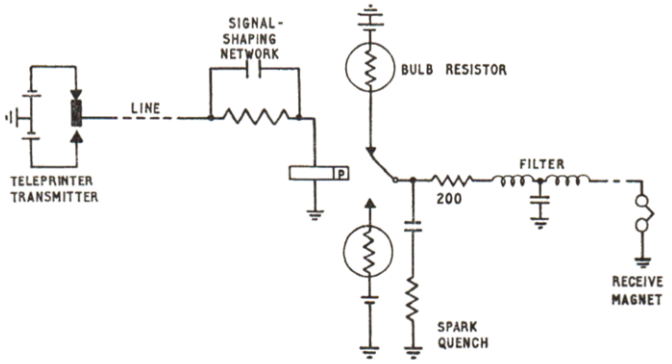
(*a*) *a single wire with earth return,*
(*b*) *a two-wire loop,*

for each direction of transmission.

Include all essential components in your diagrams.

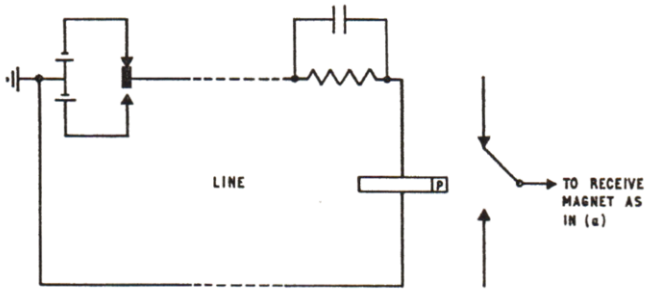
Explain clearly in what circumstances each type of circuit would be used

A. 3. The circuit diagram (*a*) shows the use of a telegraph relay working single wire with earth return. Diagram (*b*) shows a telegraph relay used with a line consisting of a 2-wire loop.



(*a*)

If telegraph signals are applied to a line, the value of the current in the distant receiver may be affected by the line constants resistance, capacitance, inductance and, if overhead circuits are used, leakance. These factors attenuate the received voltage and affect the build-up of current in the receiver. As the length of the line is increased, the effect of these factors increases, and the steady-state current may not be reached in the receiver before the next signal element affects the current. When this occurs, the response of the receiver is no longer uniform and the signal is distorted; when the amount of distortion exceeds the margin of the receive machine, the received characters are multilated. With further increase in the length of line, the receiver may fail to respond to any of the transmitted signals. To overcome this, a sensitive telegraph relay may be inserted in the centre section of the line; the relay will respond to the weak signals and will reconstitute each signal as a square wave. The relay will not, however, correct any distortion of the signal. Relays may be used for lines consisting of 20 lb/mile conductors between about 40 and 90 miles in length, operating at a speed of 50 bauds.



(*b*)

A single-wire earth-return circuit is unbalanced and is subject to interference from neighbouring power or telecommunication circuits. If the voltage induced by the interfering circuit is of sufficient magnitude, spurious signals of sufficient strength may be generated to give errors in the received telegraph message. The effect is significant in circuits exceeding 40 miles in length, and cannot be tolerated in circuits exceeding 90 miles in length. This type of interference may be eliminated by using the loop circuit shown in sketch (*b*). Neighbouring circuits induce equal e.m.f.s in the same direction in each wire of the loop and these e.m.f.s. cancel each other out.

Q. 4. *With the aid of a block diagram explain the fundamental differences between a "frequency-division" (e.g. MCVF) and a time-division telegraph system.*

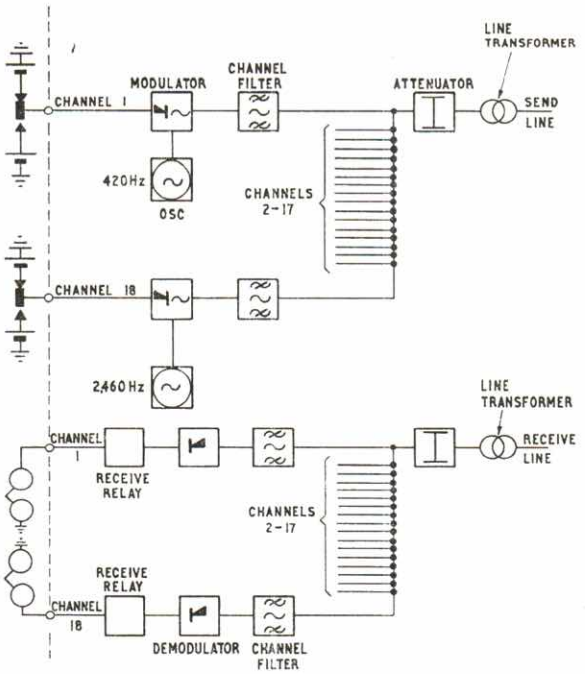
Which system is the more suitable for providing circuits for a switched telegraph network? Give reasons to support your answer.

A. 4. The block diagrams show (*a*) a frequency-division telegraph system, and (*b*) a time-division telegraph system.

A modern telephone channel has an effective bandwidth of 300 Hz to 3,400 Hz, and may be used for the transmission of a number of simultaneous telegraph signals. Band-pass filters are used to divide the frequency spectrum to allow a bandwidth of 120 Hz to each of 24 telegraph channels. The carrier frequency of each channel is modulated by the telegraph transmitter, each separate channel using a different carrier frequency. At the receiver, a band-pass filter again separates each channel frequency, and the demodulator drives an output relay to operate the teleprinter receive magnet.

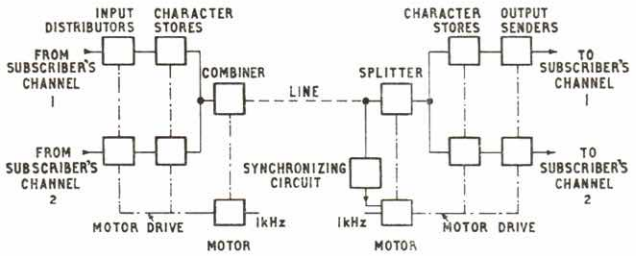
A time-division telegraph system allows each of a number of channels the exclusive use of the transmission path for a short fixed time. The send and receive ends of the transmission path operate in synchronism

to identify the signals on each particular channel. Signals from each channel are received on an input distributor and the characters stored; the combiner equipment transmits characters or elements from each



(*a*)

store at regular intervals and in cyclic order, and transmits the signals to line. At the distant termination, the splitter equipment, operating in synchronism with the combiner equipment, selects the elements or characters pertinent to each channel, and passes them to the appropriate store. The output sender transmits the signals, complete with start and stop elements, to the output channel. As the system is synchronous, the start element and stop element for each character are not required to be transmitted over the synchronous path; this permits a more rapid rate of character transmission within the available bandwidth. The start and stop elements are re-inserted at the receive equipment before the character signals are transmitted to the receive teleprinter.



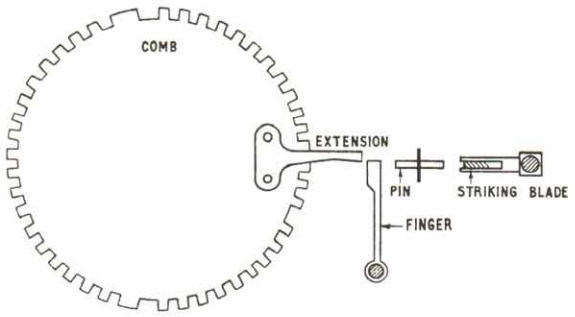
(*b*)

The frequency-division system is more suitable for a switched telegraph network because it is flexible and adaptable in application and does not require channels in tandem to operate in synchronism. Channels may be added or withdrawn, as required, and each channel may be worked independently of any other channel, providing that the transmitting apparatus is within the signalling-speed limit imposed by the bandwidth. This system may be provided more cheaply, than the time-division system, if the existing telephone network is used. Where high-cost circuits are involved, such as on radio or submarine-cable networks, the additional cost of time-division equipment may be economically justified.

***Q.* 5.** *With the aid of sketches describe the operation of the combination head and typehead clutch of a teleprinter. Explain what happens when the same character is received twice in succession.*

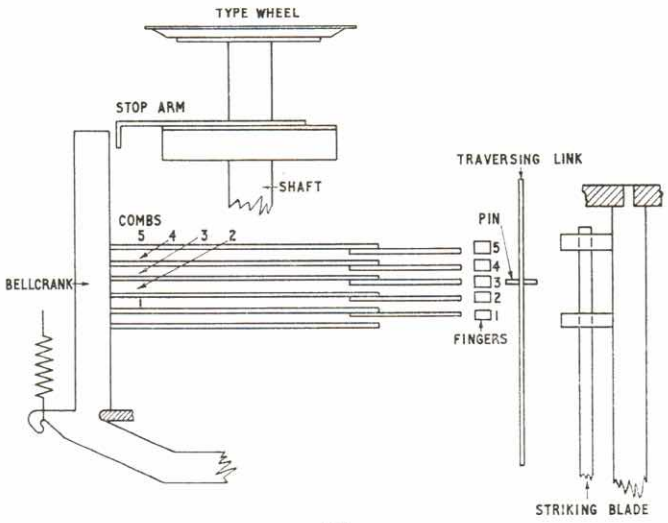
A. 5. The combination head of a teleprinter provides for the conversion of an incoming electrical signal into mechanical selection of the appropriate character in the printing mechanism. The combination head consists basically of five receiving combs, one of which is shown in sketch (*a*).

When the start element of a character is received by the teleprinter, the receiving mechanism causes the striking blade to move towards the pin once during each of the character elements. The striking blade is also controlled by the electromagnet is such a way that if the element



(*a*)

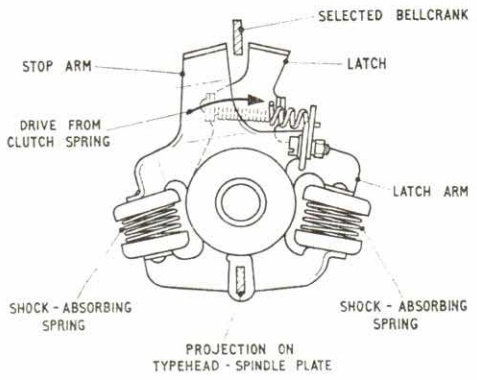
received is of start polarity the blade tilts and does not make contact with the pin; if an element of stop polarity is received, the striking blade causes the pin to move the finger to a position underneath the comb-extension. As the pin moves across the five comb-extensions, the five fingers are positioned according to the character elements received, and the combination of the incoming character is stored. Sketch (*b*) shows a plan of the combination head, the comb-setting fingers, the traversing link and the finger-setting blade.



(*b*)

Bell-cranks, corresponding to each character to be printed, are positioned around the periphery of the combination head. Under the action of a cam, the bell-cranks are lifted free of the combs just before a second cam causes the comb-setting fingers to rise. In their upward movement, according to the code combination set up, certain fingers will engage with the extensions of the receiving combs, causing them to rotate through a few degrees. The periphery of each comb is cut in such a manner that for every single combination there is only one set of slots in alignment. Hence, when the bell-cranks return to normal under the action of the return springs, one bell-crank will fall into the selected set of slots.

A continuously-rotating shaft carrying the type wheel passes through the centre of the combs. When the bell-cranks are lifted to allow the selection of the succeeding character, the typehead clutch is released and the typehead rotates. A stop arm attached to the type wheel rotates with its end just clear of the bell-cranks when these are normal. When one of the bell-cranks falls into a set of slots, the stop arm is



(*c*)

arrested by the bell-crank and the type-wheel stops with the character appropriate to the selected bell-crank in the printing position.

Sketch (*c*) shows a modern typehead clutch.

The clutch is driven continuously by contact between the clutch spring and the typehead clutch drum, and rotates in the direction shown by the arrow. As the clutch rotates, the sloping edge of the latch comes into contact with the selected bell-crank. The latch is depressed against its spring and the typehead continues to rotate until the stop-arm comes into contact with the bell-crank. The latch-arm spring causes the latch to return to its normal position after passing the bell-crank, so that the latter is positioned between the stop-arm and the latch-arm. The inertia of the typehead causes it to continue to rotate, and the centralizing pin, which is connected to the typehead, thrusts against the bottom of the latch-arm and compresses the shock-absorbing spring. The centralizing pin oscillates for a brief period between the ends of the stop-arm and the latch-arm until the energy of the moving typehead has been dissipated; the typehead is then firmly held in position by the bell-crank and the centralizing pin so that the appropriate character can be printed.

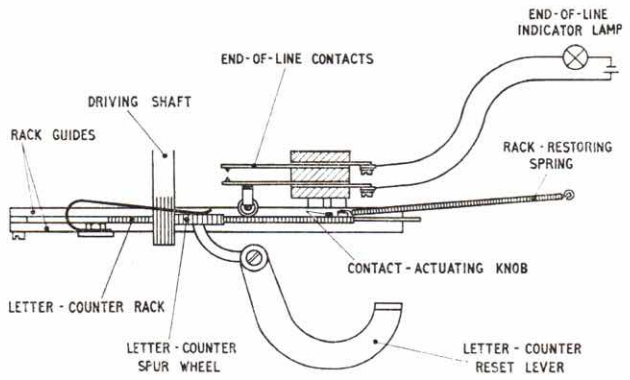
When the same character is received twice in succession, the complete cycle of operations is repeated for the second character in the same way as if a different character were received.

***Q.* 6.** *Expain with the aid of sketches.*

(a) how a page teleprinter can interwork with a tape teleprinter.

(b) how the local answer-back unit is prevented from operating when the FIGS and WRU keys are operated on a teleprinter which is arranged to provide a local record of transmitted signals.

A. 6. A page teleprinter can interwork satisfactorily with a tape teleprinter providing some reminder can be given to the operator of the tape teleprinter that the paper carriage on the page teleprinter is approaching the end of the line and that a carriage-return signal followed by a line-feed signal must be sent. This facility is offered on the tape teleprinter by providing a carriage-return indicator, consisting of a switch which may be used to operate an alarm lamp or bell.



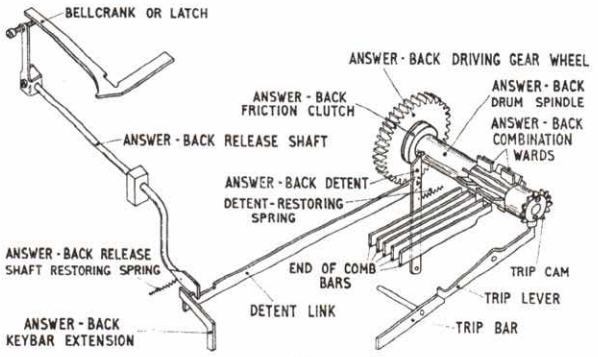
(*a*)

A sketch of a carriage-return indicator is shown at (*a*).

The mechanism on the tape teleprinter consists of a carriage-return indicator spur-wheel, driven through a pinion cut on the platen-driving shaft. The spur-wheel engages a small toothed-rack which is free to move to the left against a restoring spring. As each character is transmitted, the rack moves to the left until a small projection, located at its extreme right-hand end, deflects the indicator-switch springs; the operation of the contacts gives the end-of-line alarm. A lamp jack and 6-volt lamp fitted to the tape reel give a visible alarm, the power supply for the lamp being obtained via a jack fitted near the teleprinter and connected by means of a cord and plug.

When the alarm operates, the tape-teleprinter operator depresses the keys CARRIAGE-RETURN and LINE FEED, causing the distant page-teleprinter carriage to be returned and rotated one line space in readiness for the next line to be printed. The indicator rack is restored to normal under the action of the restoring spring by the movement of the carriage-return indicator switch-lever. The latter operates when a carriage-return signal is received (as a local record), and causes the spur wheel to move out of engagement with the rack, allowing the rack to move. The spur-wheel is re-engaged with the rack by the action of the spur-wheel engagement-spring. The alarm lamp is extinguished by the restoration of the rack.

The mechanism controlling the answer-back unit is shown in sketch (*b*). The unit is controlled by the operation of the bell-crank appropriate to the secondary character to the letter D, which operates whenever this combination is received by the teleprinter. When a teleprinter is operating with a local record, each character transmitted from the teleprinter also actuates the receive mechanism. To prevent the local answer-back unit from operating when the secondary character to the letter D is transmitted to the distant teleprinter, an extension is fitted



(*b*)

to the WRU keybar which prevents the answer-back release shaft moving the detent link and releasing the answer-back drum. The local operator maintains the pressure on the WRU key until the first character of the distant answer-back is received, this causes the bell-crank to the secondary character to the letter D to re-set and the local answer-back unit remains inactive.

***Q.* 7.** *Explain what is meant by the following services and state for what purpose each is used: (a) telex, (b) printergram, (c) gentex, (d) phonogram, (e) telephone-telegram.*

A. 7.

(*a*) *Telex*

The telex system is similar in many respects to the public telephone service except that inter-communication is effected by teleprinters instead of telephones. The basic equipment consists of subscribers' apparatus, exchange equipment and circuits connecting the subscriber to the exchange and interconnecting exchanges. The subscriber's equipment consists of a teleprinter and a control unit, the latter including a dial for selection of the distant subscriber. Circuits between the subscriber and the exchange may be physical circuits in telephone cables, m.c.v.f. circuits or a combination of both. The exchange employs 2-motion selectors in a similar manner to a telephone exchange. Printed service-signals indicate the progress of the call, and the automatic return of the distant subscriber's answer-back gives assurance that the connexion has been satisfactorily established.

The telex service offers interconnexion between any two subscribers on the system. All calls within the United Kingdom and to many foreign countries are fully subscriber dialled, with automatic charging. Access to other countries is obtained through the international telex switchboard, where calls are controlled manually.

The signalling speed is fixed at 50 bauds and the C.C.I.T.T.* Alphabet No. 2 is the standard.

(*b*) *Printergrams*

The printergram service enables telex subscribers to send and receive telegrams to and from a centre connected to the public telegraph network. Printergram centres are normally located in a teleprinter automatic switching (t.a.s.) zone centre and are available by dialling the code 01. The incoming telegram is received at a teleprinter in the printergram centre and processed as a telegram received in the normal way.

For telegrams sent from non-telex sources, the message is routed to the printergram centre appropriate to the destination of the telegram. The message is passed to the printergram operator, who dials the number of the required telex subscriber and transmits the message to the subscriber's teleprinter.

(*c*) *Gentex*

Gentex is the name given to the automatic switching network used for handling public telegrams between European countries. The system provides through transmission of messages between principal towns and cities, and avoids costly and time-wasting re-transmissions. In many countries the service is integrated with the telex network, but, in the United Kingdom, gentex centres have been set up in London and various zone centres. Incoming gentex traffic is routed over the telex network to tape teleprinters; outgoing traffic is routed through the international telex exchange in London, but is segregated from ordinary telex traffic to enable the correct charging conditions to be applied.

(*d*) *Phonograms*

The phonogram service enables telegrams to be dictated from a subscriber's telephone or a public call-box. The phonogram centre is normally located at a t.a.s. group centre. A call may be made to a phonogram centre by dialling the appropriate code or by asking the operator for the phonogram service. The call is directed to a phonogram position, causing a calling lamp to light. The phonogram

* C.C.I.T.T.—International Telegraph and Telephone Consultative Committee.

operator answers the call, using a plug and cord, and types the dictated telegram directly on to a telegram form; the telegram is then forwarded to a teleprinter position for onward transmission. In small offices, the operator may operate both the incoming phonogram suite and the outgoing teleprinter position.

Telegrams dictated from the delivery office to a subscriber's telephone are also known as phonograms.

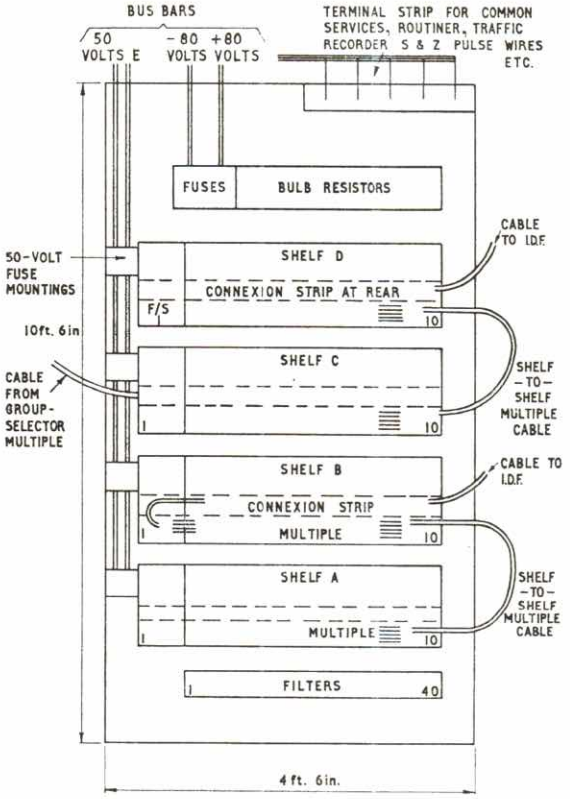
(*e*) *Telephone-Telegram Service*

Telephone-telegram working provides for the transmission of telegrams from Post Offices to the t.a.s. group centre. The public telephone network is used unless there is sufficient volume of traffic for a private circuit to be provided. Telegrams may also be transmitted in a similar manner between the group centre and the delivery office.

At the group centre, switchboards similar to the printergram positions are provided, where the telegram is typed on a telegraph form and distributed within the office for onward transmission.

***Q.* 8.** *Sketch the general arrangement of an apparatus rack suitable for housing final selectors. Indicate on your sketch the quantity and type of selectors fitted and the methods of providing multiple cabling, incoming cabling and battery feeds.*

A. 8. The general arrangements of a typical apparatus rack suitable for housing final selectors is shown in the sketch. The rack is 10 ft 6 in high and 4 ft 6 in wide, and houses 40 selectors in four shelves of 10 selectors on each shelf. Overhead bus-bars supply the 50-volt and



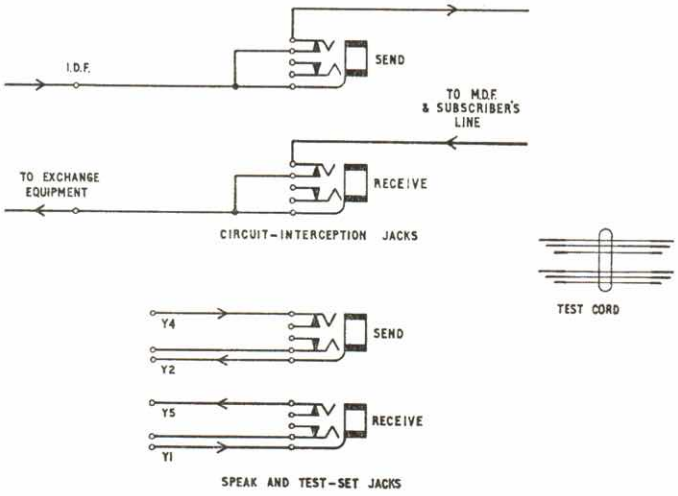
80-volt power to the rack, and these supplies are distributed through the fuse mountings; bulb resistors are also provided for the 80-volt supplies. A shelf of low-pass filters is provided at the lower end of the rack; one filter is associated with each final selector. Cables to a connexion strip at the top of the rack connect the miscellaneous common services, such as the S and Z pulse wires, or routiner connexions.

The selector bank-wiring is multipled along each shelf, with cables connecting shelves in pairs; cables lead from each pair of shelves to the intermediate distribution frame and thence to the racks of subscribers' line-circuit equipment and the subscribers' lines.

The relay-set section of each final selector is connected to the rack wiring through a plug and jack (U-point) connexion on the shelf. From each final-selector jack, multiple wires are provided for the power leads, service-signal and alarm leads; there are individual connexions to the filters and for the routiner-access points, and connexions from the group-selector multiple outlets.

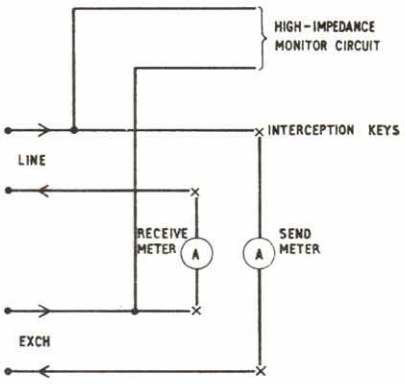
***Q.* 9.** *With the aid of diagrams show how a monitor and test set is used for testing a telex subscriber's circuit. Show the main connexions of the test set and explain clearly the reason for the importance of terminating the test set before intercepting the line circuits.*

A. 9. Sketch (*a*) shows the method of connecting a monitor-and-test set (more usually known as a speaker-and-test set) to an engineering control board (e.c.b.) to enable tests to be made on telex sub-



(*a*)

scribers' circuits. At a telex exchange all subscribers' circuits are routed through the e.c.b., using the circuit-interception jacks; the speaker-and-test set also terminates on similar jacks. To prevent interruption of the circuit and mutilation of any message being transmitted on that circuit, all the keys on the test set must be in the normal position before testing commences; the main tester connexions are then as shown in sketch (*b*), with the line wires connected to the exchange wires through current meters. The double-ended test cord at the e.c.b. is first inserted into the speaker-and-test jacks and then into the appropriate circuit-interception jack. The latter consists of a sleeve and tip connexion only, again to prevent a working circuit being interrupted.



(*b*)

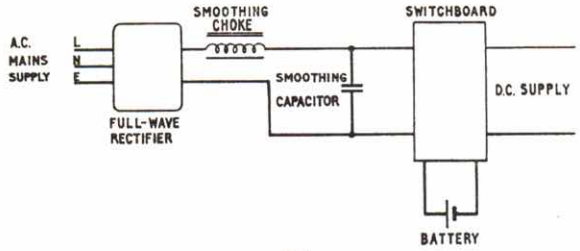
With the speaker-and-test set connected, and the line conditions observed by reading the current meters, test-set keys may be operated to select the tests to be made either on the subscriber's line and equipment or on the exchange equipment. A high-impedance monitoring circuit is provided so that signals on an engaged line may be recorded on the teleprinter associated with the test set. The test set is also capable of testing the call, dial and clear functions of the exchange and subscriber's equipment and provides for connecting distortion-measuring equipment and test-message signals to the line under test.

***Q.* 10.** *Describe a rectifier-type power plant for a large telegraph office. Illustrate your answer with a block diagram. Why are batteries included in the power plant and how is the required capacity determined?*

A. 10. The main elements of a rectifier power plant for a large telegraph office are shown in sketch (*a*).

A full-wave rectifier is connected to the mains supply, with a smoothing choke and capacitor in the output to eliminate ripple in the direct-current output. Rectifiers of the Westat type incorporate the smoothing components within the rectifier case, and are also designed to give a constant-voltage output together with automatic overload-protection.

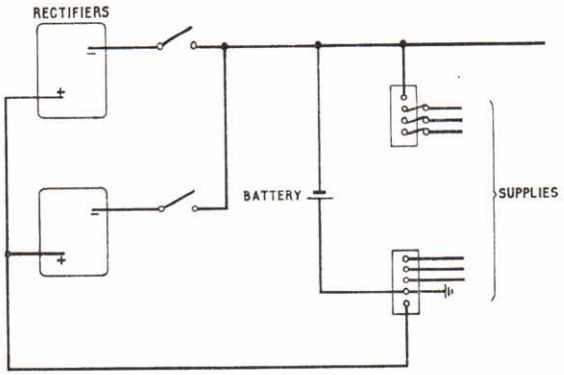
This type of rectifier, with static, automatic voltage-regulation, is most suitable for a telegraph office with teleprinter machines imposing a rapidly-varying load on the ±80-volt supply.



(*a*)

A switchboard may or may not be provided, depending on the size of the installation. The purpose of such a board is to house the connexions between the battery, the rectifier, the signalling earth and the d.c. output, and to provide a mounting for any necessary circuit-breakers, change-over switches and mains fuses. Only the equipment required for switching, and the meters for indicating voltage and current, are mounted on the front of the board; the bus bars, wiring and connexion terminals are mounted on the back of the switchboard to avoid accidental mishandling.

Batteries are included in the power plant to provide a safeguard in the event of mains failure. Machine telegraphy is much more vulnerable to brief interruptions of power supply than is telephony, as even a very short break in the supply will cause errors in the telegraph message. The battery may be provided on the principle of single-battery float operation or divided-battery float operation. In the former system the battery is supplied to give the required ampere-hour capacity for emergency use; for the divided-battery system, two batteries are provided, each battery having about 0·55 of the capacity of the single float battery. A schematic circuit diagram of a single-battery float system is shown in sketch (*b*).



(*b*)

The required capacity of the battery may be determined by several factors. The first requirement is to decide upon the length of time for which the battery will have to supply the load with the mains supply ineffective. The normal time is about 6 hours. The full load required from the battery during this time will then have to be calculated, taking into account all the machines, signalling and miscellaneous equipment which will be connected during the period of emergency. If a divided float system is to be used, one battery will be fully charged and the other discharged by 10 per cent due to losses on floating duty; thus the available capacity will be 1·9 times the capacity of each individual battery. Again, some account must be taken of the fact that battery capacity is normally specified on a 9-hour or 10-hour discharge rate; at the higher rate proposed, the cells will be less efficient. The limits of the terminal voltage for the plant must also be considered, and the battery capacity should be such that these limits are not exceeded during the discharge period. Some factor must also be used to allow for growth in the load during the life of the battery in order that the required capacity will be available when required.

RADIO AND LINE TRANSMISSION B, 1968

Students were expected to answer any six questions

Q. **1.** *Explain what is meant by the terms*

(*a*) *negative feedback,*
(*b*) *automatic gain control* (*a.g.c.*).

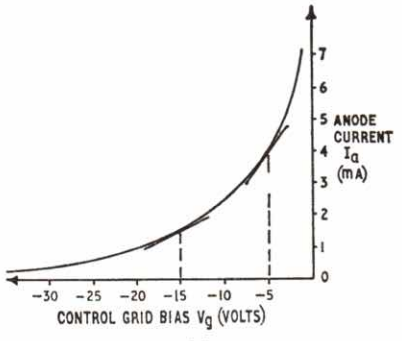
Why is negative feedback used in line amplifiers for multi-channel telephony and a.g.c. in high-frequency radio systems?

A. 1. (*a*) For the part of this question which deals with negative feed-back, see A. 5, Radio and Line Transmission B, 1967, *Supplement*, Vol. 61, p. 15, Apl. 1968.

(*b*) *Automatic Gain Control*

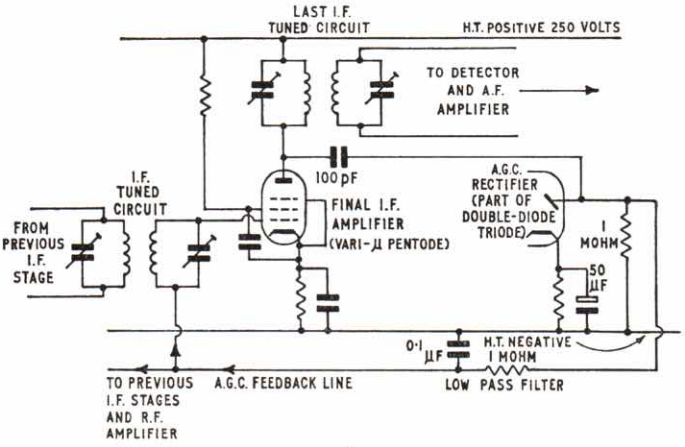
Automatic gain control (a.g.c.) should not be confused with negative feedback. An a.g.c. circuit is included in a receiver or amplifier to maintain a reasonably constant output power despite large changes in the input level. Unlike negative feedback, the feedback signal in the case of a.g.c. does not follow the instantaneous variations in the output signal, but changes proportionally to the average output power. Thus, the feedback-signal is a varying d.c., and in radio receivers it is usually derived by rectifying the i.f. output, and passing the resulting signal through a capacitor-resistor circuit having a sufficiently long time-constant to pass only slowly-varying components. This d.c. feedback is then used to supplement the bias on one or more of the preceding stages and, in order to achieve a.g.c., each such stage must be designed so that its gain varies with bias.

In a valve stage this is done by varying the pitch of the wire spiral forming the control-grid, to produce a vari-μ input characteristic (sketch (*a*)). For −15 volts bias the g_m of the valve (and hence its μ, since $\mu = r_a g_m$) is less than when the bias is −5 volts. Thus, if the feedback is connected so that increases in the i.f. output level cause the negative grid-bias to increase, any tendency for an increase in output power is counteracted by a corresponding decrease in the gain of the vari-μ stage—and vice versa.



(*a*)

Sketch (*b*) shows how these principles are applied to a typical superheterodyne receiver circuit.



(*b*)

In transistor circuits, control of gain with d.c. feedback is obtained by making use of the variation in the input resistance of a junction transistor with bias. The input characteristic, (base-current plotted against base-voltage), has a similar shape to the vari-μ valve transfer-characteristic given in sketch (*a*). If the transistor to which a.g.c. is applied is fed from a low-impedance source, then its input signal current is determined largely by the slope of its input characteristic. Thus, as the base bias-current is varied, the load on the previous stage is varied, and hence the gain of the previous stage changes.

High-frequency radio systems employ ionospheric propagation for

long distance communication. Transmissions via this medium are subject to fading, as described in A. 10, Radio and Line Transmission B, 1966, *Supplement*, Vol. 60, p. 6, Apl. 1967. The use of a.g.c. can provide a reasonably constant receiver-output under most fading conditions. Where deep fades are common, however, diversity reception is necessary.

Q. 2. *A zero-loss communication circuit has a uniform amplitude-frequency response over a bandwidth of* 48 *kHz* (*kc/s*). *A noise-free test signal having* 30 *per cent amplitude modulation is transmitted and the output signal-to-noise ratio found to be* 60 *dB.*

Calculate the new output signal-to-noise ratio for each of the conditions if:

(*a*) *a second identical communication circuit is connected in tandem with the first,*

(*b*) *a* 6 *dB attenuator is connected at the output end of the line,*

(*c*) *the depth of modulation is changed to* 60 *per cent,*

(*d*) *a filter having a rectangular passband* 24 *kHz wide, wholly within the circuit passband and including the test signal, is connected at the output end,*

(*e*) *a bandstop filter is connected between the source of the test signal and the communication circuit to eliminate one of the side-bands of the test signal.*

A. 2. (*a*) Suppose that, in a communication circuit, noise is generated, for example, by thermal agitation of electrons in a particular resistor. Subsequent to this point, both signal and noise will be subject to the same gains and losses, so the ratio of signal-power to the noise-power from this resistor remains unchanged.

Since the first circuit has zero loss, the signal level at the input to the second circuit will be equal to the level at the input to the first circuit. Therefore, because the two circuits are identical, there will be a corresponding resistor in the second circuit at which the signal level will equal that at the first resistor. This second resistor, however, will add to the signal an equal amount of noise to that already contributed by the first resistor. Considering all noise-generating sources, it follows that, at the output to the second circuit, the signal-power is the same as at the first circuit output, while the noise-power has doubled.

Hence, if suffix 1 denotes the original output conditions, and suffix 2 denotes the new conditions:

$$10\log_{10}\left(\frac{S_2}{N_2}\right) = 10\log_{10}\left(\frac{S_1}{2N_1}\right) = 10\log_{10}\left(\frac{S_1}{N_1}\right) - 10\log_{10}2,$$
$$= 60 - 3,$$
$$= \underline{57\text{ dB}}.$$

(*b*) Assuming that the attenuator itself is noise-free, then both signal and noise at the circuit output are attenuated by 6 dB, so the signal-to-noise ratio remains unchanged.

$$\therefore\ 10\log_{10}\left(\frac{S_2}{N_2}\right) = 10\log_{10}\left(\frac{S_1}{N_1}\right),$$
$$= \underline{60\text{ dB}}.$$

(*c*) A communication circuit may be assumed to produce an intelligible output, and the description of the test signal indicates that this circuit makes use of amplitude-modulation. The term output thus refers to a point in the circuit after demodulation. The output signal is therefore a replica of the input modulating signal. Therefore, an increase in modulation depth from 30 per cent to 60 per cent represents a doubling of the input and output voltages. The output noise, of course, remains unchanged.

Hence, if R_0 represents the output resistance of the circuit, and V_1 and V_2 the output voltages at the two modulation depths:

$$10\log_{10}\left(\frac{S_2}{N_2}\right) = 10\log_{10}\left(\frac{V_2^2/R_0}{N_2}\right) = 10\log_{10}\left(\frac{(2V_1)^2/R_0}{N_1}\right),$$
$$= 10\log_{10}2^2 + 10\log_{10}\left(\frac{V_1^2/R_0}{N_1}\right),$$
$$= 6 + 10\log_{10}\left(\frac{S_1}{N_1}\right),$$
$$= \underline{66\text{ dB}}.$$

(*d*) Over a bandwidth as small as 48 kHz, the noise-power per unit bandwidth due to thermal sources is constant. Thus the total noise-power N, measured in a given bandwidth B, is given by:

$$N = AB,$$

where A is a constant.

The filter described reduces the circuit bandwidth by a factor of 2,

$$\text{i.e. } B_2 = \frac{B_1}{2}.$$

$$\text{Therefore, } N_2 = AB_2 = A\frac{B_1}{2},$$
$$= \frac{N_1}{2}.$$

For testing purposes the modulating signal may be assumed to be sinusoidal (although credit would be given for another assumption if correctly argued) and thus the filter passband wholly includes the output signal and does not affect its power.

Hence,

$$10\log_{10}\left(\frac{S_2}{N_2}\right) = 10\log_{10}\left(\frac{S_1}{N_1/2}\right) = 10\log_{10}2 + 10\log_{10}\left(\frac{S_1}{N_1}\right),$$
$$= 3 + 60,$$
$$= \underline{63\text{ dB}}.$$

(*e*) The amplitude modulation of a carrier by a sine wave is described in A. 9, Radio and Line Transmission B, 1966, *Supplement*, Vol. 60, p. 6, Apl. 1967, where the instantaneous voltage (e) of the modulated wave is shown to be given by:

$$e = E_c\left[\sin\omega_c t + \frac{K}{2}\cos(\omega_c - \omega_m)t - \frac{K}{2}\cos(\omega_c + \omega_m)t\right] \quad (i)$$

where E_c is the amplitude of the carrier wave, $K = \frac{E_m}{E_c}$, where E_m is the amplitude of the modulating signal, and ω_c and ω_m are the corresponding angular frequencies.

Although there are many varieties of demodulator, the basic principle is always the same, in that a non-linear device is required (even in the linear-demodulator). The output–input characteristic of any non-linear device may be represented by a power series,

$$e_{out} = a + be_{in} + ce_{in}^2 + de_{in}^3 + \ldots \qquad \ldots\ldots(ii)$$

where a, b, c, etc. are constants, and, in this case, e_{in} is given by equation (*i*).

When equation (*ii*) is expanded, a large number of different frequency products appear, but only even-order terms (i.e. ce_{in}^2, ee_{in}^4, etc.) produce outputs at the modulating frequency (ω_m). All the unwanted frequencies are then suppressed by a filter. In many demodulators the squared term predominates, and for the double sideband input the output voltage at modulation frequency is then given by $cE_c^2K\sin\omega_m t$. When a band-stop filter is inserted to remove one sideband, however, the input voltage becomes $E_c\left[\sin\omega_c t \pm \frac{K}{2}\cos(\omega_c \pm \omega_m)t\right]$, for which the output voltage becomes $cE_c^2\frac{K}{2}\sin\omega_m t$. Thus the ratio of output signal voltages before and after inserting the filter is:

$$\frac{V_2}{V_1} = \frac{1}{2}.$$

$$\therefore \text{ the power ratio is } \frac{S_2}{S_1} = \frac{1}{4}.$$

A very narrow filter bandwidth is all that is necessary to remove the single frequency sideband, and furthermore, the filter may be assumed to occur prior to the addition of the bulk of the noise. The output noise is therefore virtually unaffected.

$$\text{Thus, } 10\log_{10}\left(\frac{S_2}{N_2}\right) = 10\log_{10}\left(\frac{S_1/4}{N_1}\right) = 10\log_{10}\left(\frac{S_1}{N_1}\right) - 10\log_{10}4,$$
$$= 60 - 6 = \underline{54\text{ dB}}.$$

Q. 3. *A class-A r.f. amplifier stage has a load circuit comprising an inductor of* 100 μH *in parallel with a loss-free capacitor of* 200 *pF. The effective Q-factor of the inductor is* 56·25.

Calculate the frequency of resonance and the frequencies of the upper and lower 3 *dB points,* f_1 *and* f_2.

Three such stages are connected in tandem. If the output voltage at resonance is 10 *volts, calculate the output voltage at* f_1 *and* f_2.

Calculate the value of resistor that would have to be connected in series with the inductor to make the 3 *dB bandwidth of the single stage amplifier equal to* 40 *kHz* (*kc/s*).

A. 3. For a high-Q parallel tuned-circuit, the frequency of resonance (f_0) is given by

$$f_0 = \frac{1}{2\pi\sqrt{LC}}\text{ Hz},$$

where L and C are respectively the inductance in Henries and the capacitance in Farads. Provided the rest of the circuit does not add significant reactance, this is also the resonant frequency of the amplifier as a whole.

$$\therefore f_0 = \frac{1}{2\pi\sqrt{100 \times 10^{-6} \times 200 \times 10^{-12}}},$$

$$= \underline{1{\cdot}125 \text{ MHz.}}$$

Also $B = \frac{f_0}{Q}$, where B is the 3 dB bandwidth of the amplifier, and Q is substantially the same as the inductor Q-factor, assuming that the rest of the amplifier does not load the tuned circuit significantly.

$$\text{Hence } B = \frac{1{\cdot}125 \times 10^6}{56{\cdot}25} = 20 \text{ kHz.}$$

But since the amplifier frequency response is symmetrical about f_0,

$$\text{then } f_1 = f_0 - \frac{B}{2}, \text{ and, } f_2 = f_0 + \frac{B}{2}.$$

$$\therefore \underline{f_1 = 1{\cdot}115 \text{ MHz}}, \text{ and, } \underline{f_2 = 1{\cdot}135 \text{ MHz}}.$$

If the output voltage at resonance from three stages in tandem is 10 volts, then at f_0, the output voltage for the first stage is $\sqrt[3]{10}$ volts.

Now at f_1 or f_2 the first stage output power has fallen to half its value at resonance, so the output voltage has fallen to $\frac{\sqrt[3]{10}}{\sqrt{2}}$ volts.

The output voltage from stage 3, at f_1 or f_2, is therefore $\left(\frac{\sqrt[3]{10}}{\sqrt{2}}\right)^3$, or $\underline{3{\cdot}54 \text{ volts.}}$

For the inductor, $Q = \frac{2\pi f_0 L}{r}$, where r is the coil series-resistance,

$$\therefore r = \frac{2\pi f_0 L}{Q} \text{ ohms.}$$

Now when a resistor of value R ohms is connected in series with the coil, a new Q-factor emerges, i.e. $Q' = \frac{2\pi f_0 L}{r + R}$.

But Q' is also given by $\frac{f_0}{B'}$, where B' is 40 kHz.

$$\text{Thus, } r + R = \frac{2\pi f_0 L}{Q'} = \frac{2\pi f_0 L}{f_0/B'},$$

$$= 2\pi L B'.$$

$$\therefore R = 2\pi L B' - r = 2\pi L B' - \frac{2\pi f_0 L}{Q},$$

$$= 2\pi L\left(B' - \frac{f_0}{Q}\right),$$

$$= 2\pi L(B' - B),$$

$$= \underline{12{\cdot}6 \text{ ohms.}}$$

Q. 4. *Draw a block diagram of a superheterodyne general purpose communication receiver and briefly explain the function of each stage.*
Explain the principal advantages of adding an r.f. amplifier stage.

A. 4. For the answer to the first part of this question see A. 2, Radio and Line Transmission B, 1966, *Supplement*, Vol. 60, p. 2, Apl. 1967.

The principle advantages of adding an r.f. amplifier stage are:

(*i*) As explained in the first part of the answer, the r.f. amplifier tuned circuits are designed to reject the image channel.

(*ii*) The frequency-changer stage, by virtue of its construction, is very noisy. If no r.f. amplifier were provided the signal at the frequency-changer input would be at its lowest level, and thus the noise level would be appreciable compared to the signal. If some amplification is included in front of the frequency-changer, however, the signal will be stronger at the point where the worst noise is encountered. This amplification is provided by the r.f. stage, which is capable of being designed to generate much less noise than the frequency-changer. The signal-to-noise ratio at the receiver output is thereby greatly improved.

(*iii*) The r.f. amplifier also acts as a buffer stage to help prevent the local-oscillator output reaching the receiver aerial, and thus being radiated to cause interference to other sources.

Q. 5. *Define the decibel.*
An amplifier has input and output impedances of 75 ohms resistive. When a potential difference of 0·5 volts is applied across the input a potential difference of 10 volts appears across the output.
Calculate:
(*a*) *the voltage-gain ratio,*
(*b*) *the power-gain ratio,*
(*c*) *the gain in decibels.*
If the output impedance is changed to 150 ohms resistive, the input impedance and voltage-gain ratio being unaltered, calculate:
(*i*) *the power-gain ratio,*
(*ii*) *the gain in decibels.*

A. 5. The British Standard definition for the decibel (Term number 12021, B.S. 204, 1960) is "A unit of transmission giving the ratio of two powers. One-tenth of a Bel. The number of decibels is given by

$$N = 10 \log_{10}\left(\frac{P_1}{P_2}\right),$$

where P_1 and P_2 represent two values of power."

Note.—If the gain of an amplifier, for example, is required to be expressed in decibels, the power fed to the amplifier and the power delivered to the load must be evaluated and the logarithm of the ratio of the two multiplied by 10. The same result will not be produced by evaluating twenty times the logarithm of the ratio of output and input voltages, unless the load and input resistances are equal.

Let P_0, V_0, and R_0 be output power (in watts), voltage (in volts), and resistance (in ohms), respectively.

Let P_i, V_i, and R_i be input power, voltage and resistance respectively. Then:

(*a*) Voltage-gain ratio $= \frac{V_0}{V_i} = \frac{10}{0{\cdot}5} = 20.$

(*b*) Power-gain ratio $= \frac{P_0}{P_i} = \frac{V_0^2/R_0}{V_i^2/R_i}$

$$= \left(\frac{V_0}{V_i}\right)^2 \frac{R_i}{R_0},$$

$$= 20^2 \times \frac{75}{75}, = \underline{400}.$$

(*c*) The gain in decibels $= 10 \log_{10}\left(\frac{P_0}{P_i}\right) = 10 \log_{10}(400),$

$$= 10 \times 2{\cdot}6,$$

$$= \underline{26 \text{ dB.}}$$

(*i*) With the output impedance changed to 150 ohms resistive, but the voltage gain unchanged,

$$\text{Power-gain ratio} = \left(\frac{V_0}{V_i}\right)^2 \frac{R_i}{R_0} = 400 \times \frac{75}{150},$$

$$= \underline{200.}$$

(*ii*) Gain in decibels $= 10 \log_{10}\left(\frac{P_0}{P_i}\right) = 10 \log_{10}(200),$

$$= 10 \times 2{\cdot}3,$$

$$= \underline{23 \text{ dB.}}$$

[Note that this is not the same as $20 \log_{10}\left(\frac{V_0}{V_i}\right)$].

Q. 6. *Draw a block diagram showing the apparatus required for a 4-channel independent sideband h.f. radio transmitter.*
It is common practice to use reduced or suppressed carriers in both independent sideband h.f. radio systems and in multi-channel line transmission systems. Discuss the reasons for doing this in each case.

A. 6. For the answer to the first part of this question, see A. 1, Radio and Line Transmission B, 1967, *Supplement*, Vol. 61, p. 13, Apl. 1968. The reasons for using reduced or suppressed carriers in independent sideband h.f. radio systems are also discussed in the same reference.

In multichannel line transmission systems, however, the reasons are rather different. Ideally every stage in the system should have a completely linear transfer-function, i.e. every change in input voltage or current should produce an exactly proportional change in output voltage or current. Practical stages, however, will exhibit small departures from linearity, and this is especially true of stages handling the largest signals (e.g. the output stages of line amplifiers), since these have to operate over a wide range of their output characteristics. This results in intermodulation between the various voice signals comprising the multichannel block, which becomes intolerable for even small non-linearities.

If the signal to be handled can be reduced to a minimum, while still producing a useful output, the intermodulation will also be minimized. Each carrier component contains no useful information but contributes significantly to the loading of the amplifier, i.e. to the total signal power. By suppressing the carriers, therefore, no useful signal is lost but the multichannel intermodulation is considerably reduced.

Q. 7. *Make careful sketches on squared paper showing the appearance of the following:*

(a) An r.f. carrier, whose amplitude when unmodulated is 1 volt peak, amplitude modulated by a 1-kHz (kc/s) sinewave tone. The maximum amplitude of the modulated envelope is 2·8 volts (peak-to-peak). Use scales of 1 in = 0·5 ms and 1 in = 1 volt.

(b) The output of a double-balanced modulator. Assume that the envelope of the output signal has an amplitude of 1 volt peak and the modulating frequency is 1 kHz. Use scales of 1 in = 0·5 μs and 1 in = 1 volt.

(c) A train of rectangular pulses of width 1 μs, pulse repetition frequency 100 kHz and amplitude 2 volts. Use scales of 1 in = 5 μs and 1 in = 1 volt.

For the waveform defined in (a), what is the depth of modulation?

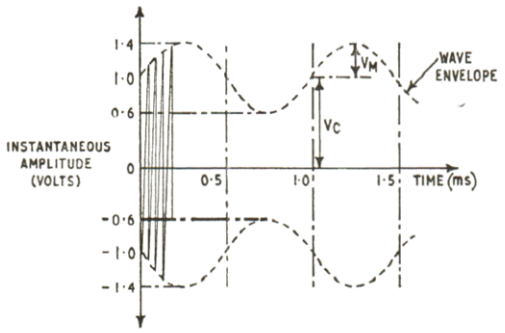
For the waveform defined in (c), sketch the amplitude-frequency distribution of the spectrum.

A. 7. (*a*) The frequency of modulation, and hence of the envelope of the modulated wave, is 1 kHz.

$$\therefore \text{ Period of envelope} = \frac{1}{\text{frequency}} = \frac{1}{1{,}000} \text{ second},$$

$$= 1 \text{ ms}.$$

Sketch (*a*) shows the waveform.



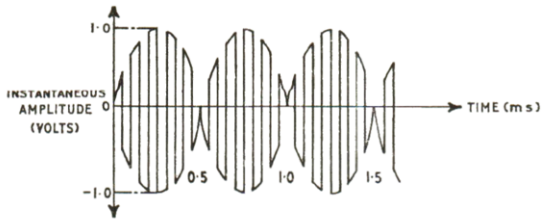
(*a*)

$$\text{The depth of modulation} = \frac{V_m}{V_c} = \frac{0{\cdot}4 \text{ volts}}{1{\cdot}0 \text{ volt}},$$

$$= 0{\cdot}4,$$

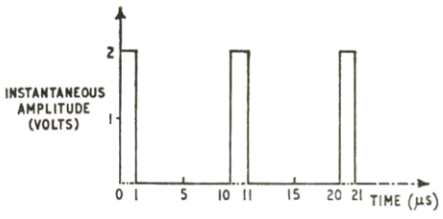
$$= \underline{40 \text{ per cent.}}$$

(*b*) Under conditions of no modulation a double-balanced modulator produces zero output, the carrier wave being suppressed in the output transformer due to cancellation of fluxes. The period of the modulation is again 1 ms (see sketch (*b*)).



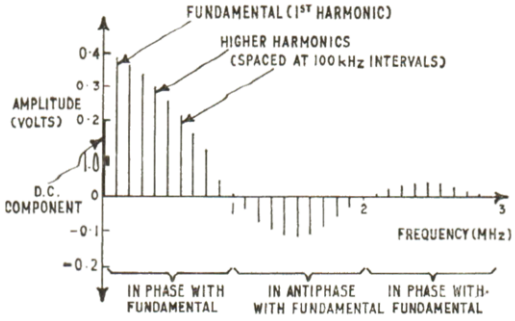
(*b*)

(*c*) Pulse repetition frequency (p.r.f.) = 100 kHz, or 10^5 Hz. Therefore, the interval between similar points on successive pulses is $1/10^5$ seconds, or 10 μs (see sketch (*c*)). This pulse train can be analyzed mathematically into an infinite series of sine waves of increasing



(*c*)

frequency which, if all plotted on the same voltage–time axes and added at each point in time, will be found to result in the waveform shown in sketch (*c*). The amplitude–frequency spectrum is shown in sketch (*d*), and it should be noted that the sinusoidal components



(*d*)

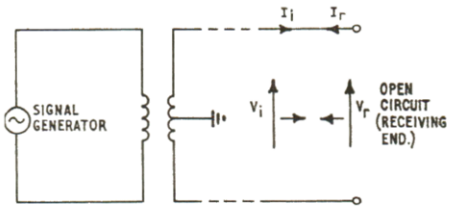
(often referred to as spectral lines) occur at frequencies which are multiples of the pulse repetition frequency. The sine wave whose frequency equals the p.r.f. is called the fundamental. Spectral lines of zero amplitude occur at frequencies equal to $\frac{1}{\tau}, \frac{2}{\tau}, \frac{3}{\tau}$ etc., where τ is the pulse width in seconds. Between these frequencies all lines are alternately in phase and in antiphase with the fundamental component at the centre of each pulse.

Q. 8. *Sketch the distribution of current and voltage along a half-wave dipole aerial in free space and explain briefly how this distribution arises.*

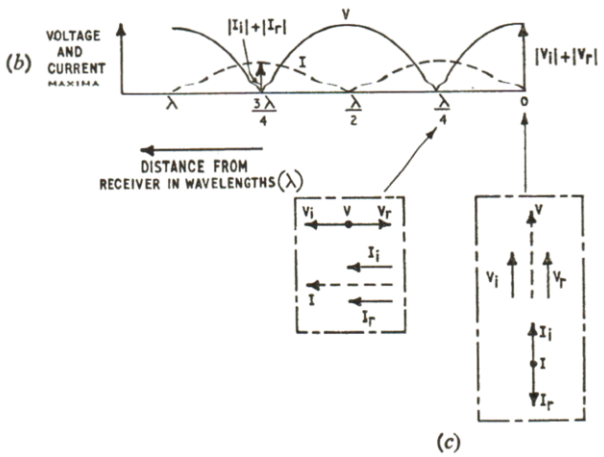
Hence, demonstrate that the aerial presents a low impedance to the feeder.

A dipole aerial requires to be fed with 20 kW of power to produce a given signal strength at a particular distant point. If the addition of a reflector makes the same field strength available with an input power of only 11 kW, what is the gain in decibels obtained by the use of the reflector?

A. 8. Sketch (*a*) shows a typical open-circuited parallel-wire transmission line and sketch (*b*) shows the distributions of voltage and current maxima along this transmission line from the receiving end. Sketch (*c*) shows the vector diagrams at two points along the transmission line.



(*a*)



(*c*)

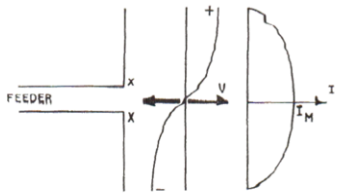
Energy sent down the line from the generator is reflected by the open circuit, and the voltage distribution shown can be considered as the sum of incident and reflected components (V_I and V_r, respectively). The current distribution may be similarly analyzed.

At a perfect open circuit, $|V_I| = |V_r|$, and the two components add, so doubling the voltage. The net current at the open circuit, however, must clearly be zero, and this results because I_r is equal to I_I at this point, but flows in the reverse direction.

At a distance $\frac{\lambda}{4}$ back from the receiving end, the vectors representing the incident voltage and current will correspond to the instantaneous

values transmitted a quarter period later in time than the components at the open circuit, i.e. delayed by 90° in phase. The vectors representing the reflected components, however, will correspond to values transmitted earlier, i.e. advanced in phase by 90° relative to components at the open circuit.

Since the transmission line is shown balanced about earth, the voltage of one wire is positive with respect to earth at the open circuit, whilst that of the other is negative with respect to earth. If the wires comprising the line are opened out at the point $\frac{\lambda}{4}$ back from the receiving end, a half-wave dipole aerial is formed, and the distribution of voltage relative to earth potential is shown in sketch (*d*). The consequent current distribution is also shown.



(*d*)

The transmission-line feeder is connected to the dipole at "X–X," and the impedance at this point $\left(\frac{V}{I}\right)$ appears to be zero ohms. A practical dipole, however, will exhibit losses, and the actual impedance offered to the feeder depends upon frequency, length, aerial construction, element resistance, leakage to earth, etc. Because of these many factors involved, the exact impedance is difficult to predict, so the aerial is normally adjusted experimentally to present a resistive impedance of about 80 ohms to the feeder.

The power flux-density (P) at a point d metres from the dipole aerial of gain G_1 (relative to some reference, e.g. an isotropic radiator) is given by:

$$P = \frac{W_1 G_1}{4\pi d^2} \text{ watts/metre}^2,$$

where W_1 is the power fed to the aerial.

P is also proportional to the square of the field strength (E) at distance d, therefore, since E remains unchanged when the reflector is added, P also remains constant.

Thus if W_2 and G_2 are the power fed to the aerial and its gain (relative to isotropic) in the presence of the reflector,

$$P = \frac{W_2 G_2}{4\pi d^2} \text{ watts/metre}^2.$$

$$\therefore \quad W_2 G_2 = W_1 G_1,$$

$$\text{and} \qquad \frac{G_2}{G_1} = \frac{W_1}{W_2}.$$

Hence, the gain in decibels obtained by the use of the reflector is:

$$\text{Gain} = 10 \log_{10}\left(\frac{G_2}{G_1}\right) = 10 \log_{10}\left(\frac{W_1}{W_2}\right),$$

$$= 10 \log_{10}\left(\frac{20}{11}\right),$$

$$= \underline{2{\cdot}6 \text{ dB}}.$$

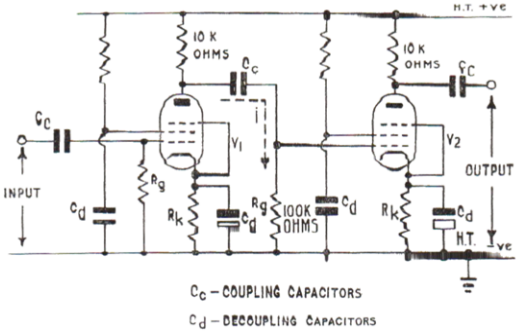
Q. 9. *Draw a circuit diagram of a two-stage audio-frequency valve amplifier operating in class-A with resistance-capacitance coupling.*

Each valve has an anode a.c. resistance of 20,000 *ohms and an amplification factor of* 100 *and the anode load resistors are* 10,000 *ohms. Calculate the voltage gain of the amplifier.*

If the grid resistor of the second valve is 100,000 *ohms, calculate the value of coupling capacitor that will cause the voltage gain at* 50 *Hz (c/s) to be one-half the gain at* 1 *kHz. Assume that the reactance of the coupling capacitor at* 1 *kHz is negligible and that the influence of other factors on the low-frequency response is negligible.*

A. 9. The circuit diagram described is shown in the sketch. In each valve under no-signal conditions the direct anode current (I_a) flows through the resistor of value R_k, thus developing a potential difference $I_a R_k$ which makes the cathode positive with respect to earth. The control-grid is connected to earth via resistor R_g, and since this electrode does not emit electrons, grid-current cannot flow to develop a similar voltage across R_g. The grid is, therefore, virtually at earth potential, and negative with respect to the cathode. In this way negative-bias is applied to each valve, and the grid–cathode bias-voltage (e.g. that voltage producing class-A conditions) is largely determined by the value of R_k.

Resistor R_g also provides a leak path for the few electrons, from



C_c – COUPLING CAPACITORS

C_d – DECOUPLING CAPACITORS

the cathode, which are intercepted by the grid wires, so preventing the progressive build-up of negative charge on the grid.

The voltage gain (m) of each stage is given by the expression:

$$m = \frac{\mu R_L}{r_a + R_L},$$

where μ is the voltage amplification factor,
r_a is the anode a.c. (slope) resistance,
and R_L is the net load resistance fed by the valve.

Since the resistor R_g has a high value and the grid of the second valve takes no current when negatively biased, the second stage of the amplifier may be assumed to have negligible shunting effect on the anode load of the first valve (V1). Also, it may be assumed that the amplifier output also feeds a high resistance. Therefore R_L may be taken as 10 kohms for both valve stages.

If a small alternating voltage, of amplitude v, is applied to the amplifier input, the output of the first stage will be a voltage of amplitude mv. This voltage is applied to the second stage input and is then subject to a further gain m. This produces an output from the second stage of amplitude $m^2 v$.

$$\text{Thus, the amplifier voltage gain} = \left(\frac{\mu R_L}{r_a + R_L}\right)^2,$$

$$= \left(\frac{100 \times 10{,}000}{20{,}000 + 10{,}000}\right)^2,$$

$$= (33{\cdot}3)^2,$$

$$= \underline{1111}.$$

Consider the coupling components, C_c and R_g, connecting the anode of valve V1 to the control-grid of the second valve V2. At 1 kHz, since the reactance of capacitor C_c is negligible, virtually the whole alternating component of valve V1 anode-voltage (v_a) from the first stage is applied between the grid of valve V2 and earth. At 50 Hz, however, the reactance of capacitor C_c is sufficient to cause an appreciable potential difference to appear across this capacitor, thus reducing the input voltage to valve V2 and thereby lowering the amplifier gain.

Note that the intrinsic gains of the two stages remain unaltered, but the interstage coupling becomes less efficient.

The voltage gain will fall to half that at 1 kHz when the alternating p.d. across resistor R_g is half the alternating anode-voltage of the first stage.

Let i be the current through the series combination of C_c and R_g.
Let C_c be the capacitance of capacitor C_c, and R_g be the resistance of resistor R_g.

$$\text{Then, } i = \frac{v_a}{\text{series impedance of } C_c \text{ and } R_g} = \frac{v_a}{\sqrt{R_g^2 + \left(\frac{1}{2\pi f C_c}\right)^2}},$$

where f is the signal frequency.

$$\therefore \text{ p.d. across resistor } R_g = iR_g = \frac{v_a R_g}{\sqrt{R_g^2 + \left(\frac{1}{2\pi f C_c}\right)^2}}.$$

Now at 1 kHz, $\frac{1}{2\pi f C_c}$ is negligible,

$$\text{therefore, } iR_g = v_a,$$

$$\text{but at 50 Hz, } iR_g = \frac{v_a}{2}.$$

$$\therefore \quad \frac{v_a R_g}{\sqrt{R_g^2 + \left(\frac{1}{2\pi f C_c}\right)^2}} = \frac{v_a}{2}.$$

Rearranging this equation and cancelling v_a gives:

$$C_c = \frac{1}{2\sqrt{3}\pi f R_g} \text{ F},$$

$$= \frac{10^6}{2\sqrt{3}\pi 50 \times 10^5}\ \mu F,$$

$$= \underline{0{\cdot}0184\ \mu F.}$$

Q. 10. *Sketch the arrangement of a 4-tube coaxial cable for inland use. Show typical dimensions and mention the materials used.*

Why is each tube lapped with steel tape?

A coaxial cable whose dielectric loss is negligible has an attenuation of 2·9 dB/mile at 0·5 MHz (Mc/s). Calculate the frequency at which the loss of a six-mile length will be 54 dB.

A. 10. For the answer to the first part of this question see A. 5, Radio and Line Transmission B, 1966, *Supplement*, Vol. 60, p. 3, Apl. 1967.

Each tube is lapped with steel-tape to provide low-frequency screening. This helps to prevent crosstalk between tubes up to a frequency of about 100 kHz. It is not included to give the cable mechanical strength.

The cable attenuation (α) at high frequencies is given by $\alpha = K\sqrt{f}$ dB/mile, where f is the frequency in Hz and K is a constant of proportionality.

$$\therefore \text{ At } 0{\cdot}5 \text{ MHz}, \quad K\sqrt{0{\cdot}5 \times 10^6} = 2{\cdot}9.$$

$$\therefore K = \frac{2{\cdot}9}{\sqrt{0{\cdot}5 \times 10^6}}.$$

At frequency f' a loss of 54 dB for a 6-mile length of cable represents an attenuation (α') of 9 dB/mile.

Therefore, $\alpha' = K\sqrt{f'}$.

$$\therefore f' = \left(\frac{\alpha'}{K}\right)^2 = \left(9 \times \frac{\sqrt{0{\cdot}5 \times 10^6}}{2{\cdot}9}\right)^2,$$

$$= 4{\cdot}81 \times 10^6 \text{ Hz},$$

$$= \underline{4{\cdot}81 \text{ MHz}}.$$

TELECOMMUNICATION PRINCIPLES C, 1968

Students were expected to answer any six questions

Q. 1. *Explain briefly why power is dissipated in the dielectric of a capacitor when it is connected across an alternating supply.*

A sinusoidal voltage is applied to a capacitor which has a small loss. Sketch a phasor (vector) diagram for the capacitor voltage and current showing clearly the loss angle.

A sinusoidal voltage of 40 volts r.m.s., at a frequency of 1·59 kHz (kc/s), is applied to a 1 μF capacitor which has a loss angle of 2×10^{-4} radian.

Calculate:

(a) the power dissipated in the capacitor,
(b) its equivalent shunt resistance,
(c) its equivalent series resistance.

A. 1. For the first part of the answer see A.1. Telecommunication Principles C, 1966, *Supplement*, Vol. 60, p. 24, Apl. 1967.

(*a*)

The phasor (vector) diagram is given in sketch (*a*). The current vector OI is drawn at an angle, ϕ, in advance of the voltage vector, OV. The angle ϕ is less than 90° by an angle ψ, which is the loss angle.

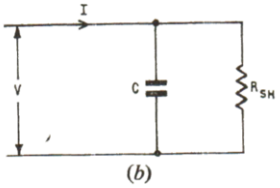
Since the voltage applied to the capacitor is sinusoidal, the current supplied will be sinusoidal. Also, the loss angle is very small, therefore,

$$\psi \simeq \sin\psi = \sin(90 - \phi),$$
$$= \cos\phi,$$
$$= \text{power factor.}$$

The supply current $I = \omega CV = 2\pi \times 1{\cdot}59 \times 10^3 \times 1 \times 10^{-6} \times 40,$
$= 0{\cdot}4$ amps.

$\therefore$ Power dissipated in the capacitor $W = VI\cos\phi,$
$= 40 \times 0{\cdot}4 \times 2 \times 10^{-4},$
$= \underline{3{\cdot}2 \text{ mW}}.$

If R_{SH} is the equivalent shunt resistance, see sketch(*b*),



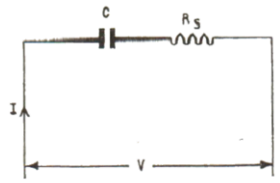
(*b*)

then, $W = \dfrac{V^2}{R_{SH}},$

or, $R_{SH} = \dfrac{V^2}{W} = \dfrac{40^2}{3{\cdot}2 \times 10^{-3}},$

$= \underline{0{\cdot}5 \text{ Mohms}}.$

Let R_S be the equivalent series resistance, see sketch (*c*)



(*c*)

then, $W = I^2R_S,$

or, $R_S = \dfrac{W}{I^2} = \dfrac{3{\cdot}2 \times 10^{-3}}{0{\cdot}4^2},$

$= \underline{0{\cdot}02 \text{ ohm}}.$

Q. 2. *Fig. 1 is an equivalent circuit for an iron-cored transformer.*

Explain the significance of the components r, L_1, R, L_2 and C in this equivalent circuit.

Describe an experimental method of determining the frequency response of such a transformer between a resistive load. Sketch a typical response curve.

Briefly discuss the frequency response with reference to the equivalent circuit.

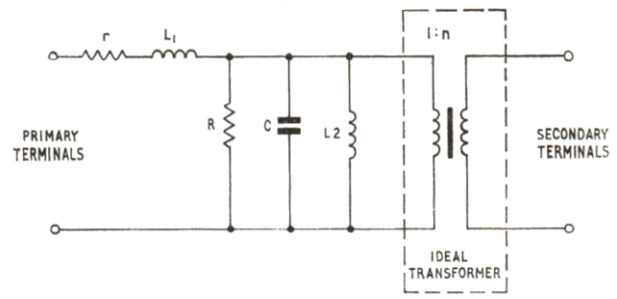


Fig. 1

A. 2. In the given equivalent circuit,

r = the winding resistances referred to the primary,

$$= R_P + \frac{1}{n^2}R_S.$$

L_1 = the winding leakage inductances referred to the primary,

$$= L_P + \frac{1}{n^2}L_S.$$

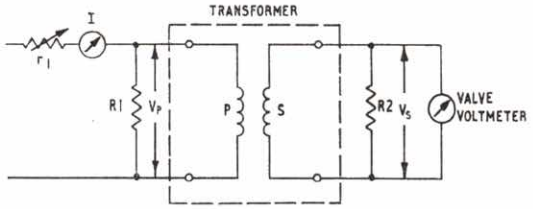
C = the winding capacitances referred to the primary,

$$= C_P + n^2C_S.$$

R represents the primary shunt resistance such that $\dfrac{V^2}{R}$ gives the core losses (due to hysteresis and eddy current losses).

L_2 is the effective inductance of the primary winding with the secondary circuit open-circuited such that $\frac{V2}{\omega L_2}$ gives the magnetizing current.

In one method of determining the frequency response of a transformer, a low resistance R_1, is connected across the primary winding of the transformer and a variable-frequency sinusoidal-source is connected in series with an ammeter and variable resistance r_1, across the transformer as in sketch (*a*).

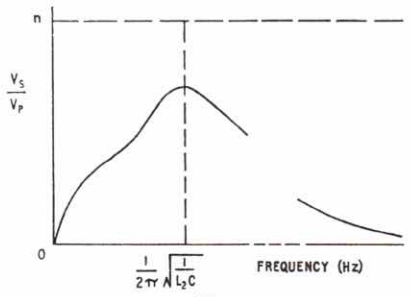


(*a*)

The input voltage $V_P = IR_1$.

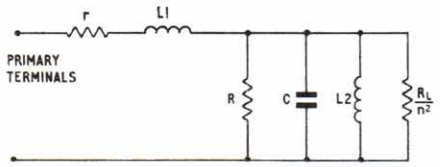
The load resistance, R_1, is connected across the secondary winding and a valve voltmeter is connected across it to obtain voltage V_S. Voltages V_P and V_S are measured at various frequencies, with the current I being maintained approximately uniform by varying r_1.

The ratio $\frac{V_S}{V_P}$ at various frequencies is plotted against frequency, see sketch (*b*).



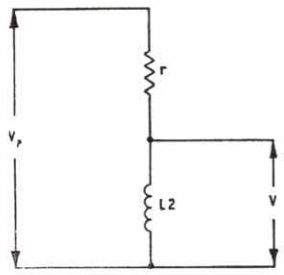
(*b*)

The primary circuit of the loaded transformer can be represented as in sketch (*c*). At very low frequencies the primary circuit approxi-



(*c*)

mates to sketch (*d*) so that the ratio of $\frac{V}{V_P}$ and, therefore, $\frac{V_S}{V_P}$ is small.



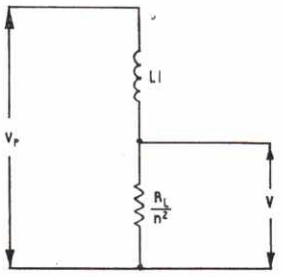
(*d*)

At the resonant frequency of C and L_2,

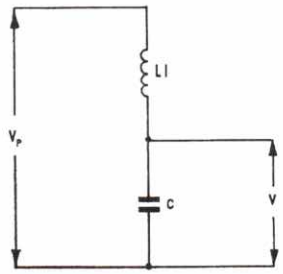
$$f_0 = \frac{1}{2\pi\sqrt{CL_2}},$$

and the primary circuit can be represented as in sketch (*e*). $\frac{R_L}{n^2}$ should be very much greater than $\omega_0 L_1$, and hence $\frac{V_S}{V_P} \to n$.

At very high frequencies the primary circuit approximates to sketch (*f*). Again the ratio $\frac{V}{V_P}$ and, therefore, $\frac{V_S}{V_P}$ is very small.



(*e*)



(*f*)

Q. 3. *State Thévènin's theorem.*

Deduce the Thévènin equivalent circuit for terminals AB *in the circuit of Fig.* 2.

Hence calculate the magnitude of the current flowing in an impedance $(200 - j100)$ *ohm when it is connected across* AB.

What impedance will take maximum power from terminals AB *and what will be the value of this power?*

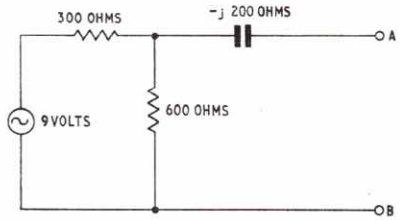


Fig. 2

A. 3. *Thévènin's Theorem.* The current in a load impedance connected across two terminals, AB, of a network of impedances and generators, is the same as if this load were connected across a simple constant-voltage generator whose e.m.f. is the open-circuit voltage measured across AB, and whose internal impedance is the impedance of the network looking back into the terminals AB, with all generators replaced by impedances equal to their internal impedances. The voltage across the terminals AB when they are open circuited is:

$$\frac{600}{600 + 300}\ 9 = 6 \text{ volts.}$$

The impedance measured across the terminals AB when the generator is replaced by a short circuit is:

$$\frac{300 \times 600}{300 + 600} - \text{j } 200 = (200 - \text{j } 200) \text{ ohms.}$$

Thus, the Thévènin equivalent circuit for the terminals AB of the given network comprises a generator having an e.m.f. of 6 volts and an internal impedance of 200 − j 200 ohms.

When an impedance of (200 − j 100) ohms is connected across AB, the current supplied from the network is:

$$\frac{6}{200 - \text{j } 100 + 200 - \text{j } 200} = \frac{6}{400 - \text{j } 300}.$$

$$\therefore\ |I| = \frac{6}{500} = \underline{12 \text{ mA.}}$$

An impedance of (200 + j 200) ohms will take maximum power from the network since the circuit will then be non-reactive. For this condition, maximum power is delivered to the load when its resistance is equal to the internal resistance of the equivalent generator.

The maximum power will be

$$VI = \frac{6}{2} \times \frac{6}{400} = \underline{45 \text{ mW.}}$$

Q. 4. *A 3-winding transformer bridge is set up as shown in Fig. 3.*

A 1,500-pF capacitor in parallel with a 1-kohm resistor is connected across XY. Calculate the values of C1 and R1 for no output at the detector when the secondary winding AB has

(a) the same number of turns as winding BC,
(b) ten times the number of turns as winding BC.

Show how the bridge circuit of Fig. 3 should be modified, without using additional components, to measure an inductive admittance connected across XY. Explain why the balance condition would then be frequency dependent, even when the unknown admittance has a constant shunt inductance.

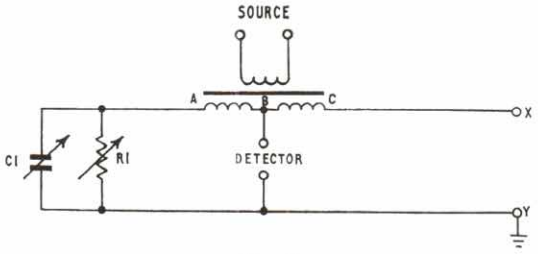


Fig. 3

A. 4. (*a*) $C = 1{,}500$ pF $R = 1$ kohms.

(*b*) Since the C1,R1, combination will have ten times the voltage across the given capacitor and resistance in parallel, its impedance must be ten times that of the impedance across XY.

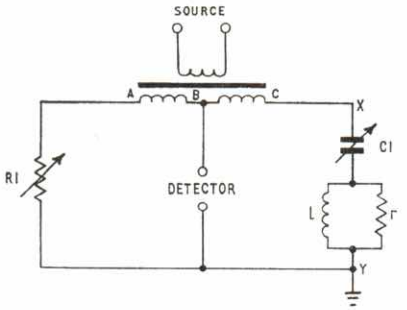
$\therefore C_1 = 150$ pF, and $R_1 = 10$ kohms.

The bridge circuit should be modified as shown in the sketch, in order to measure inductive admittances.

Assuming windings AB and BC have the same number of turns, the balance condition is

$$R_1 = \frac{1}{j\omega C_1} + \frac{j\omega lr}{r + j\omega l},$$

$$= \frac{r + j\omega l - \omega^2 l C_1 r}{j\omega C_1 r - \omega^2 l C_1}.$$



where l and r are the inductance and resistance of the inductive admittance.

Thus, $j\omega C_1 r R_1 - \omega^2 l C_1 R_1 = r + j\omega l - \omega^2 l C_1 r.$

Comparing imaginary parts gives

$$\omega C_1 r R_1 = \omega l,$$

$$\text{or, } l = C_1 r R_1. \qquad \ldots\ldots (1)$$

Comparing real parts gives

$$\omega^2 l C_1 R_1 = \omega^2 l C_1 r - r.$$

$$\therefore r\left(1 - \frac{1}{\omega^2 l C_1}\right) = R_1.$$

$\therefore$ from equation (1),

$$l = \frac{C_1 R_1^2}{1 - \dfrac{1}{\omega^2 l C_1}},$$

$$\therefore l - \frac{1}{\omega^2 C_1} = C_1 R_1^2,$$

$$\text{or, } l = C_1 R_1^2\left(1 + \frac{1}{\omega^2 C_1^2 R_1^2}\right).$$

If l is constant, both C1 and R1 must be varied as ω is varied, i.e. the balance is frequency dependent.

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